Industry Dynamics, International Trade, and Economic Growth

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This version: January 2014

Abstract

This paper theoretically investigates how international trade and dynamic trade policies affect structural change (industrialization), life-cycle dynamics of industries, and aggregate economic growth in a two-country world. Analytic solutions are obtained to establish four main results: (1) Both industry upgrading and aggregate growth are endogenously synchronized and facilitated by the investment-specific technology progress (ISTP) of the trade partner if and only if the inter-temporal elasticity of substitution exceeds unity; (2) The ISTP of the trade partner affects the starting time of industrialization of the late bloomer, but not the early bloomer; (3) Industry life spans in the early bloomer are different before and after its trade partner takes off; (4) Accelerating trade liberalization has a non-monotonic impact on aggregate output growth and industry dynamics.

Key Words: Structural Change; Industry Dynamics; International Trade; Economic Growth; Trade Policies; Intertemporal Elasticity of Substitution

JEL Codes: E21, F43, O14, O41

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1 Introduction

How is economic growth affected by international trade and trade liberalization? How is industrialization and industry dynamics affected by international trade and trade liberalization? In this paper we develop a formal model to address these two important questions in a unified and tractable framework.

The first question has been intensively studied but still far from being settled. In particular, the empirical findings are mixed. On one hand, there exist empirical evidences supporting the view that international trade and trade liberalization help increase income levels and/or growth rates (see, *e.g.*, Sachs and Warner (1995), Frankel and Romer (1999), Wacziarg and Welch (2008)). On the other hand, however, some researchers cast doubt on the legitimacy of these findings (see, *e.g.*, Rodriguez and Rodrik (2001), McMillan and Rodrik (2011)).[1]

The first goal of this paper, therefore, is to shed light on this debate by showing that the impact of trade and trade policies on economic growth can be non-monotonic. Both convergence and divergence are shown to be possible under free trade, depending on whether the trade partner has a faster investment-specific technological progress (or ISTP thereafter) *a la* Greenwood, Hercowitz and Krusell (1997). Moreover, the output growth can be facilitated or hampered by the ISTP of the trade partner, depending on whether the inter-temporal elasticity of substitution is larger than one. This is because there are two competing effects when the rate of foreign ISTP increases. One is the inter-temporal terms-of-trade effect, which tends to raise the saving rate and output growth rate because imports become increasingly cheaper over time. The second effect is the market-size expansion effect, which tends to lower the saving rate and output growth rate because the household income increases as the export market expands. These two effects exactly cancel out when the inter-temporal elasticity of substitution is equal to unity. When it is larger than unity, the inter-temporal terms-of-trade effect, which is essentially an inter-temporal substitution effect, dominates, so the output growth is facilitated by the ISTP of the foreign country, *vice versa*.

We also characterize the impact on output and consumption growth of any arbitrary dynamic tariff adjustment. The speed of trade liberalization and the magnitude of the inter-temporal substitution elasticity are both found crucial. Time-invariant tariff has no growth effect. When a home country unilaterally accelerates trade liberalization, its consumption growth rate and output growth rate both first increase and then decrease, if the inter-temporal substitution elasticity is larger than one. On the other hand, when the trade partner accelerates trade liberalization unilaterally, it always boosts the consumption growth of the home country, irrespective of the inter-temporal substitution elasticity, although the marginal effect is diminishing over time. In contrast, the impact on the output growth would turn exactly the opposite when the inter-temporal elasticity

[1] Comprehensive theoretical surveys include Grossman and Helpman (1991) and Ventura (2005). Edwards (1993) and Baldwin (2004) provide surveys on the empirical literature.

of substitution crosses the threshold value unity. These theoretical findings suggest that we may have to go beyond static trade policies to understand why the growth effect of trade liberalization is often found ambiguous in the empirical literature. Within our knowledge, this is also the first paper that highlights the importance of the inter-temporal elasticity of substitution in determining the growth impact of trade and dynamic trade policies.

The second goal of this paper is to explore the impact of trade and dynamic trade policies on industrialization (takeoff) and the life-cycle industry dynamics at a disaggregated level. Most existing literature of structural change studies the sectorial composition shift in agriculture, industry, and service (Kuznets facts) using closed-economy models. Notable exceptions include Mastuyama (1992, 2009), who shows that the impact of a sectorial productivity increase on structural change could be diametrically opposite under autarky versus in an open economy. Uy, Yi and Zhang (2013) show that trade plays a quantitatively important role in driving structural change. Distinct from these Ricardian analyses, we investigate how trade affects the economic takeoff (industrialization) in an endogenous growth model with two factors. We show how the ISTP of the trade partner affects the time to take off in the late bloomer, the country that industrializes later than its trade partner, whereas trade has no impact on when the early bloomer takes off.

Our model highlights the role of endogenous capital accumulation in driving the life-cycle dynamics of all the alternating disaggregated industries along the aggregate growth path, so it is closely related to the dynamic Heckscher-Ohlin literature. Ventura (1997) constructs a two-sector neoclassical growth model with HO trade, in which factor price equalization is ensured by construction, so the sustained return to capital incentivizes a country to maintain a high saving rate despite fast growth. By relaxing certain restrictive assumptions in Ventura (1997), Bajona and Kehoe (2010) argue that factor price equalization may not always hold and they also show quantitatively that both convergence and divergence are possible, depending on the elasticity of substitution between the traded goods. Caliendo (2011) analytically characterizes the whole dynamics in the traditional 2 by 2 HO model when the technology is Cobb-Douglas and divergence is also shown to be likely.[2]

Different from these two-sector large-country models, in our environment there are infinite industries with different capital intensities, capturing the fact that even the manufacturing sector alone covers a wide spectrum of sub-industries ranging from the labor-intensive apparels and textiles up to capital-intensive aircraft and precision equipment. This multiple-cone setting allows us to investigate the impact of trade on the endless industrial upgrading process and the equilibrium life cycle dynamics of each industry at the disaggregated level. Closed-form solutions are obtained to characterize the following pattern of industry dynamics: as capital accumulates endogenously and reaches a

[2] Growth is also studied in dynamic Hechscher-Ohlin models in a small open economy such as Findlay (1970), Stiglitz (1970), Mussa (1978), Atkeson and Kehoe (2000), Chatterjee and Shukayev (2012), Ju, Lin, Wang (2012), *etc.* However, ours is a two-large-country model, in which the endogenous terms of trade are crucial for our main results.

certain threshold, a new industry appears, booms, reaches the peak, and then declines, gradually replaced by an even more capital-intensive new industry, *ad infinitum.*[3] This hump-shaped life cycle dynamics is qualitatively consistent with the empirical facts (see, *e.g.*, Chenery *et al* (1986) and Schott (2003)).[4] Since the aggregate growth and underlying industry dynamics are endogenously synchronized in our model, the aforementioned impact of trade and trade policies on the aggregate growth is immediately applicable to industry dynamics. More precisely, industrial upgrading is facilitated by the ISTP of the trade partner, if and only if the inter-temporal elasticity of substitution exceeds unity; Accelerating trade liberalization has a non-monotonic impact on industry dynamics. Moreover, it is shown that the industry life spans are different in the early bloomer before and after its trade partner takes off.

From the methodological perspective, it is technically challenging to fully characterize the whole dynamics even for a trade model with only two sectors (see, *e.g.*, Boldrin and Deneckere (1990), Chen (1992), Caliendo (2011), Nishimura and Shimomura (2002)). Now we have infinite industries in an infinite-horizon general equilibrium world with two large countries. The form of the aggregation production function itself may change endogenously as a consequence of structural change and composition shifts in the underlying industries. It means that, mathematically, we must solve a Hamiltonian system with endogenously switching state equations subject to trade interdependence with endogenous terms of trade. Despite all these extra complications, we still obtain the analytic solution to fully characterize the whole dynamic system including the initial transitional process of industrialization and the hump-shaped life cycle dynamics of each individual industry along the aggregate growth path.

There is also a huge literature addressing the roles of innovation and technology adoption (or diffusion) in driving the industrial dynamics, product cycles, and economic growth. Krugman (1979) builds a product-cycle trade model with grow, showing that the South converges to the North if and only if the imitation speed sufficiently exceeds the innovation speed (see, among others, Flam and Helpman (1987), Grossman and Helpman (1991), Stokey (1991), Eaton and Kortum (2001)). Our model complements these studies by focusing on a distinctive driving force for industry upgrading and growth: As capital becomes more abundant, cost minimization pushes industries to upgrade toward those using capital more intensively, while the technologies for different industries are freely available within each country.[5] Ederington and McCalman (2009) study how international trade affects industrial evolution when firms make strategic dynamic decisions as the production cost exogenously decreases over time, whereas the change in the

[3] The theoretical research in the multiple-cone HO literature is mostly either purely static (see, e.g., Dornbusch, Fischer and Samuelson (1980)) or dynamic but with exogenous savings (see, *e.g.*, Leamer (1987))

[4] Ju, Lin and Wang (2012) document the data pattern of the hump-shaped industrial dynamics with the US data of the manufacturing sector at six digit industry level covering 473 industries from 1958 to 2005. The cross-country evidence on the hump-shaped industrial pattern based on the UNIDO data sets is provided in Haraguchi and Rezonja (2010).

[5] Acemoglu (2007) shows that technical change is biased toward using more abundant production factors.

production cost is the key in our model and it is endogenous.

The paper is organized as follows. Section 2 develops a static model of two-country free trade and introduces notations. Section 3 extends the static model into a dynamic environment with endogenous growth and free trade. Section 4 examines the impact of dynamic trade policies on industry life cycles and aggregate growth. The last section concludes.

2 Static Model with Free Trade

2.1 Environment

The world consists of two countries indexed by $i = 1, 2$. Each country is almost identical to the autarky environment developed in Ju, Lin and Wang (2015). There is a unit mass of identical households in each country. Each household in country i is endowed with L_i units of labor and E_i units of capital. The aggregate good of country i is produced with the following technology

$$X_i = \sum_{n=0}^{\infty} \lambda_n x_{i,n},$$

where $x_{i,n}$ denotes the output of intermediate good n in country i and λ_n is the productivity coefficient for good n, where $n \geq 0$. Each intermediate good represents an industry.[6] We require $x_{i,n} \geq 0$ for any n.

Following Acemoglu and Ventura (2002), we assume that the final consumption good in country i is determined by

$$C_i = C_{i,1}^{\alpha_i} C_{i,2}^{\beta_i}, \tag{1}$$

where $C_{i,j}$ denotes country i's consumption of the aggregate good produced by country j, and $\alpha_i \geq 0$, $\beta_i \geq 0$, and $\alpha_i + \beta_i = 1$, for $i, j \in \{1, 2\}$.[7] In particular, when $\alpha_1 = 1$ and $\alpha_2 = 0$, both countries are degenerated to autarky, which is studied in Ju, Lin and

[6] The assumption of perfect substitutability across different industries in the final output is adopted mainly for analytical simplicity, which is usual in the growth literature. For example, the agriculture Malthus production and the modern Solow production are two linearly additive components for the total output in Hansen and Prescott (2002). Also see Lucas (2009). The main qualitative features will remain valid when the substitution is imperfect, but closed-form characterization becomes infeasible. See Ju, Lin and Wang (2012) for more detailed discussions.

[7] Feenstra, Obstfeld, and Russ (2010) find that this elasticity is not significantly different from one based on the macroeconomic data. Bajona and Kehoe (2010) argue that whether convergence or divergence depends on whether the Armington trade elasticity is larger or smaller than unity. We choose the value of unity to take a neutral stand. For more discussions, refer to Shiells, Stern and Deardorff (1986 and Shiells and Reinert (1993).

Wang (2015). Let us focus on the simple case when households in both countries have the same preference: $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \beta$.[8]

Intermediate goods are only useful in the domestic production so they are not traded. Capital and labor can move freely across different industries within a country but cannot move internationally. Since the law of one price holds under free trade, there will be no trade in the final consumption good. We set the final consumption good as the numerare.

The utility function of a representative household in country i is CRRA:

$$U_i = \frac{C_i^{1-\sigma} - 1}{1-\sigma}, \text{ where } \sigma \in (0, \infty). \tag{2}$$

All the technologies exhibit constant returns to scale. In particular, one unit of labor produces one unit of good 0. For any good $n \geq 1$, the production function is Leontief:

$$F_n(k, l) = \min\{\frac{k}{a_n}, l\}, \tag{3}$$

where a_n is the capital requirement to produce one unit of good n. Sector 0 is interpreted as a traditional "Malthusian" sector, whereas all the goods $n \geq 1$ as a whole are interpreted as a modern "Solow" sector, to follow the terminology of Hansen and Prescott (2002). Factor reallocation from sector 0 to higher-indexed sectors is also referred as industrialization.

Without loss of generality, we assume a_n increases with n. Empirical evidence suggests that the productivity of the more capital-intensive intermediate inputs is generally higher (presumably as it embodies better technology), so we assume λ_n also increases in n. Therefore, a higher-indexed good has a higher productivity and is also more capital intensive. To obtain analytical solutions, we choose the following simplest parametric forms:

$$\lambda_n = \lambda^n, \ a_n = a^n, \tag{4}$$

$$\lambda > 1 \text{ and } a - 1 > \lambda. \tag{5}$$

With Leontief production functions and perfectly substitutable intermediate goods, $a > \lambda$ must be imposed to rule out the trivial case that only the good with the highest productivity is produced. Since producing good 0 does not require capital, this condition must be strengthened to $a - 1 > \lambda$.

2.2 Equilibrium

All the markets are perfectly competitive. Let P_i, r_i, and w_i denote, respectively, the price of aggregate good X_i, rental price of capital, and wage rate in country i, for $i = 1, 2$.

[8] In the appendix, we generalize the analysis by allowing α and β to be country-specific and endogenous to trade policies.

Let $p_{i,n}$ denote the price of intermediate good n in country i, where $n \geq 0$. A profit-maximizing firm in country i solves

$$\max_{x_{i,n}\geq 0}\left[P_i \sum_{n=0}^{\infty} \lambda^n x_{i,n} - \sum_{n=0}^{\infty} p_{i,n} x_{i,n}\right],$$

which yields

$$p_{i,n} = \lambda^n P_i = \begin{cases} w_i + a^n r_i, & \text{when} \quad n \geq 1 \\ w_i, & \text{when} \quad n = 0 \end{cases}. \tag{6}$$

The total income of a representative household in country i is $w_i L_i + r_i E_i$, which is equal to the total value added $P_i X_i$. A household in country i maximizes (2) subject to the following budget constraint

$$P_1 C_{i,1} + P_2 C_{i,2} = P_i X_i. \tag{7}$$

Goods markets clear internationally:

$$C_{1,1} + C_{2,1} = X_1;\ C_{1,2} + C_{2,2} = X_2.$$

It is straightforward to show

$$C_1 = \alpha X_1^{\alpha} X_2^{\beta},\ C_2 = \beta X_1^{\alpha} X_2^{\beta}. \tag{8}$$

That is, the final consumption of each country is a Cobb-Douglas aggregation of total goods produced by the two countries. Moreover, given the factor endowment $\{E_i, L_i\}_{i=1}^{2}$, there exists a unique competitive equilibrium, in which the quantities are summarized in Table 1.

Table 1: Static Trade Equilibrium

$0 \leq E_i < aL_i$	$a^n L_i \leq E_i < a^{n+1} L_i$ for $n \geq 1$
$x_{i,0} = L_i - \frac{E_i}{a}$ $x_{i,1} = \frac{E_i}{a}$ $x_{i,,j} = 0$ for $\forall j \neq 0, 1$	$x_{i,n} = \frac{L_i a^{n+1} - E_i}{a^{n+1} - a^n}$ $x_{i,n+1} = \frac{E_i - a^n L_i}{a^{n+1} - a^n}$ $x_{i,j} = 0$ for $\forall j \neq n, n+1$
$X_i = L_i + (\lambda - 1)\frac{E_i}{a} \Rightarrow$ $E_{i,(0,1)}(X_i) \equiv \frac{a}{\lambda - 1}(X_i - L_i)$	$X_i = \frac{\lambda^{n+1} - \lambda^n}{a^{n+1} - a^n} E_i + \frac{\lambda^n (a - \lambda)}{a - 1} L_i \Rightarrow$ $E_{i,(n,n+1)}(X_i) \equiv \left[X_i - \frac{\lambda^n (a-\lambda)}{a-1} L_i\right] \frac{a^{n+1} - a^n}{\lambda^{n+1} - \lambda^n}$

Proposition 1 *In the static world, there exists a unique free-trade equilibrium, in which for any country $i \in \{1, 2\}$, the industrial and aggregate output are given by Table 1, consumption C_i is given by (8), intermediate good prices $\{p_{i,n}\}_{n=0}^{\infty}$ are given by (6), and final good price P_i, wage rate w_i, rental rate r_i are given by the following, respectively:*

$$P_1 = \alpha \left(\frac{X_2}{X_1}\right)^{\beta} \text{ and } P_2 = \beta \left(\frac{X_1}{X_2}\right)^{\alpha},$$

$$w_i = \begin{cases} P_i & \text{when} \quad 0 \leq E_i < aL_i \\ \frac{\lambda^n (a-\lambda)}{a-1} P_i & \text{when} \quad a^n L_i \leq E_i < a^{n+1} L_i, \forall n \geq 1 \end{cases},$$

$$r_i = \begin{cases} \frac{\lambda - 1}{a} P_i & \text{when} \quad 0 \leq E_i < aL_i \\ \frac{\lambda^{n+1} - \lambda^n}{a^{n+1} - a^n} P_i & \text{when} \quad a^n L_i \leq E_i < a^{n+1} L_i, \forall n \geq 1 \end{cases}.$$

Proof. The proof is almost identical to that of Proposition 1 in Ju, Lin and Wang (2015). The prices are derived from (6) together with the normalization assumption for the ultimate consumption good:

$$\left(\frac{P_1}{\alpha}\right)^{\alpha}\left(\frac{P_2}{\beta}\right)^{\beta}=1,$$

and the term of trade is

$$\frac{P_1}{P_2}=\frac{\alpha X_2}{\beta X_1}, \tag{9}$$

which is derived from the balanced trade condition. ■

From this proposition, we can see that in the trade equilibrium, the aggregate output of each country is a linear function of domestic capital and labor endowments. Moreover, the functional form of the aggregate production changes when the underlying industries endogenously shift with the domestic capital-labor ratio, as shown in Figure 1.

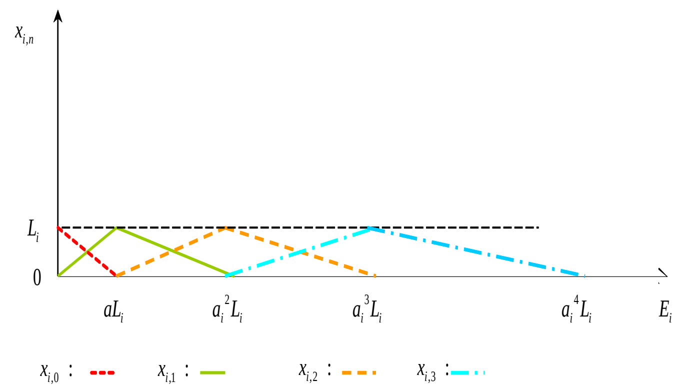

Figure 1: Industry Output in Static Trade Equilibrium in Country $i \in \{1,2\}$.

Different from the autarky equilibrium characterized in Ju, Lin and Wang (2015), now the aggregate consumption of domestic output $C_{i,i}$ and aggregate domestic output X_i are no longer necessarily equal due to international trade. The terms of trade (9) deteriorate when capital endowment becomes larger (holding labor endowment fixed). This property holds independent of the Cobb–Douglas Armington aggregation so long as the substitution elasticity between the two aggregate tradables is positive.

3 Dynamic Model with Free Trade

Now we extend the static model developed in the previous section into a dynamic one to fully characterize the industry and aggregate dynamics. Without loss of generality, we focus on the problem for country 1. By the Second Welfare Theorem, we can characterize the competitive equilibrium by resorting to the following social planner problem:

$$\max_{C_1(t)}\int_0^{\infty}\frac{C_1(t)^{1-\sigma}-1}{1-\sigma}e^{-\rho t}dt$$

subject to

$$\dot{K_1} = \xi_1 K_1(t) - E_1(X_1(t)), \tag{10}$$

$$X_1(t) = \left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)}\right]^{\frac{1}{\alpha}}, \tag{11}$$

$$K_1(0) \text{ is given,}$$

where ρ is the time discount rate, and $K_1(t)$ is the *stock* of working capital at t, which cannot be traded or used for direct consumption. At each time, the capital inherited from the past can be transformed into new working capital using the AK technology and ξ_1 is a parameter that measures the rate of the investment-specific technological progress (ISTP)(Greenwood, Hercowitz, and Krusell, 1997).[9] All the new working capital can be used to either produce the consumption good or to save (invest). $E_1(X_1(t))$ is the total capital *flow* needed to produce the aggregate good $X_1(t)$ and then fully depreciates.

The key deviation from autarky is the additional constraint (11), which comes from (8). This condition characterizes the link between the output of the two countries.

Consumption goods are non-storable. Labor endowment L_1 is constant over time. Following the pertinent literature, to ensure a positive consumption growth and to exclude the explosive solution, we impose

$$0 < \xi_i - \rho < \sigma \xi_i, \forall i = 1, 2. \tag{12}$$

Function $E_1(X_1(t))$ is a step function depending on which intermediate goods are active. In particular, $E_1(X_1(t)) \equiv E_{1,(n,n+1)}(X_1)$, which is specified in the bottom row of Table 1 for any $n \geq 0$, denoting how much capital is used to produce X_1 when only industries n and $n+1$ coexist in country 1. $E_1(X_1)$ is a strictly increasing, continuous, piecewise linear function of X_1. It is not differentiable at $X_1 = \lambda^n L_1$, for any $n \geq 0$. Therefore, the above dynamic problem may involve endogenous changes in the functional form of the state equations. That is, (10) can be rewritten as

$$\dot{K_1} = \begin{cases} \xi_1 K_1, & \text{when} \quad X_1 < L_1 \\ \xi_1 K_1 - E_{1,(0,1)}(X_1), & \text{when} \quad L_1 \leq X_1 < \lambda L_1 \\ \xi_1 K_1 - E_{1,(n,n+1)}(X_1), & \text{when} \quad \lambda^n L_1 \leq X_1 < \lambda^{n+1} L_1, \text{ for } \forall n \geq 1 \end{cases}.$$

It is verified that the objective function is strictly increasing, differentiable, and strictly concave while the constraint set forms a continuous convex-valued correspondence, hence the equilibrium must exist and be unique. The optimization problem for country 2 can be written symmetrically. For the sake of analytic simplicity, international borrowing is prohibited so that trade is balanced at each time point:

$$\beta P_1(t) X_1(t) = \alpha P_2(t) X_2(t), \forall t. \tag{13}$$

[9] A standard endogenous-growth interpretation for the AK model is that the productivity A is endogenouly determined by the amount of production as measured by the capital input, subject to decreasing return to scale. That is, $A(K) = \xi K^{\alpha}$. It captures the learning by doing. The production function for the final output is also subject to decreasing return to scale conditional on the productivity: $Y = A(K)K^{1-\alpha}$, so the total output ultimately equals ξK , ensuring sustainable growth.

For any $i = 1, 2$, let $t_{i,0}$ denote the *latest* time point when the aggregate output is equal to L_i in country i, which may be interpreted as the time to take off, or the starting time of industrialization. Output per capita grows only afterwards. Let $t_{i,n}$ denote the earliest time point when $X_i = \lambda^n L_i$ for any $n \geq 1$, which is in fact the time point when industry n reaches its peak. As the aggregate consumption $C_1(t)$ is monotonically increasing over time in equilibrium (to be verified soon), the problem can be rewritten as

$$\max_{C_1(t)} \int_0^{t_{1,0}} \frac{C_1(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt + \sum_{n=0}^{\infty} \int_{t_{1,n}}^{t_{1,n+1}} \frac{C_1(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt$$

subject to

$$\dot{K}_1 = \begin{cases} \xi_1 K_1 & \text{when} \quad 0 \leq t < t_{1,0} \\ \xi_1 K_1 - E_{1,(0,1)}(X_1), & \text{when} \quad t_{1,0} \leq t < t_{1,1} \\ \xi_1 K_1 - E_{1,(n,n+1)}(X_1), & \text{when} \quad t_{1,n} \leq t < t_{1,n+1}, \text{ for } n \geq 1 \end{cases},$$

$K_1(0)$ is given.

According to Table 1, when $t_{1,0} \leq t < t_{1,1}$, goods 0 and 1 are produced and the capital requirement function is given by $E_{1,(0,1)}(X_1) = \frac{a}{\lambda-1}(X_1 - L_1)$. When $t_{1,n} \leq t < t_{1,n+1}$ for any $n \geq 1$, goods n and $n+1$ are produced and $E_{1,(n,n+1)}(X_1) = \left[X_1 - \frac{\lambda^n(a-\lambda)}{a-1} L_1\right] \frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}$. When $K_1(0)$ is sufficiently small (to be more precise below), then there exists a period $[0, t_{1,0}]$, during which only good 0 is produced and the saving rate of capital is one. If $K_1(0)$ is sufficiently large, the economy may start with producing good $\widetilde{n}$ and $\widetilde{n}+1$ for some $\widetilde{n} \geq 1$, then $t_{1,n}$ is not defined for any $n \leq \widetilde{n}$.

3.1 Aggregate Dynamics

Define the growth rates of aggregate consumption and aggregate output of consumption goods in country i as follows, respectively:

$$\theta_i(t) \equiv \frac{\dot{C}_i(t)}{C_i(t)}; \; h_i(t) \equiv \frac{\dot{X}_i(t)}{X_i(t)}, \text{ for } i = 1, 2.$$

Proposition 2 *In the dynamic trade equilibrium,*

$$h_1(t) = \begin{cases} 0, & \textit{when} \quad t < \min\{t_{1,0}, t_{2,0}\} \\ \frac{\xi_1-\rho}{\beta+\alpha\sigma}, & \textit{if} \quad t_{1,0} \le t < t_{2,0} \\ 0, & \textit{if} \quad t_{2,0} \le t < t_{1,0} \\ \beta(\xi_1-\xi_2) + \frac{\alpha\xi_1+\beta\xi_2-\rho}{\sigma}, & \textit{when} \quad t \ge \max\{t_{1,0}, t_{2,0}\} \end{cases}, \tag{14}$$

$$h_2(t) = \begin{cases} 0, & \textit{when} \quad t < \min\{t_{1,0}, t_{2,0}\} \\ 0, & \textit{if} \quad t_{1,0} \le t < t_{2,0} \\ \frac{\xi_2-\rho}{\alpha+\beta\sigma}, & \textit{if} \quad t_{2,0} \le t < t_{1,0} \\ \alpha(\xi_2-\xi_1) + \frac{\alpha\xi_1+\beta\xi_2-\rho}{\sigma}, & \textit{when} \quad t \ge \max\{t_{1,0}, t_{2,0}\} \end{cases}, \tag{15}$$

$$\theta_1(t) = \theta_2(t) = \begin{cases} 0, & \textit{when} \quad t < \min\{t_{1,0}, t_{2,0}\} \\ \frac{\xi_1-\rho}{\frac{\beta}{\alpha}+\sigma}, & \textit{if} \quad t_{1,0} \le t < t_{2,0} \\ \frac{\xi_2-\rho}{\frac{\alpha}{\beta}+\sigma}, & \textit{if} \quad t_{2,0} \le t < t_{1,0} \\ \frac{\alpha\xi_1+\beta\xi_2-\rho}{\sigma}, & \textit{when} \quad t \ge \max\{t_{1,0}, t_{2,0}\} \end{cases}. \tag{16}$$

where $t_{1,0}$ *and* $t_{2,0}$ *are given by (27) in Lemma 2 below.*

Proof. Refer to Appendix 1. ∎

This proposition shows that the output growth rates are generally different for the two countries. For future references, let i^* (or i^{**}) denote the index of the early (or late) bloomer, the country which takes off earlier (or later) than its trade partner, where $i^*, i^{**} \in \{1, 2\}$. (14) and (15) can be rewritten in a more compact way:

$$h_{i^*}(t) = \begin{cases} 0 & \text{when} \quad t \in [0, t_{i^*,0}) \\ \widehat{h}(i^*) & \text{when} \quad t \in [t_{i^*,0}, t_{i^{**},0}) \\ \widetilde{h}(i^*) & \text{when} \quad t \in [t_{i^{**},0}, \infty) \end{cases},$$

$$h_{i^{**}}(t) = \begin{cases} 0 & \text{when} \quad t \in [0, t_{i^{**},0}) \\ \widetilde{h}(i^{**}) & \text{when} \quad t \in [t_{i^{**},0}, \infty) \end{cases},$$

where

$$\widehat{h}(i^*) \equiv \frac{\xi_{i^*} - \rho}{-1 + i^* - (-1)^{i^*}\beta + [2 - i^* + (-1)^{i^*}\beta]\,\sigma}, i^* \in \{1, 2\}, \tag{17}$$

$$\widetilde{h}(i) \equiv (2 - i - \alpha)(\xi_1 - \xi_2) + \frac{\alpha\xi_1 + \beta\xi_2 - \rho}{\sigma}, \text{ for } i \in \{1, 2\}. \tag{18}$$

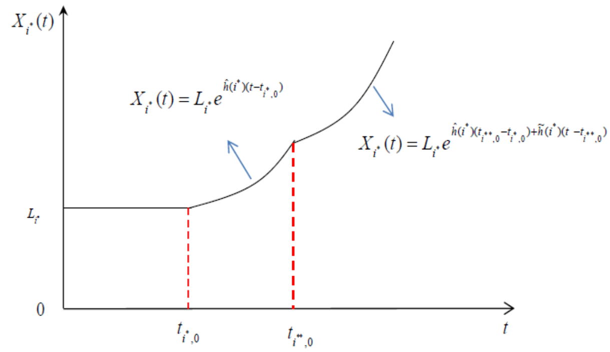

Figure 2. Time Path of Output X in Early Bloomer i^*

From Figure 2, we can see that country i^* switches from the Malthusian regime with stagnant output to the Solow regime at time point $t_{i^*,0}$, after which the output grows. Notice that, before the trade partner takes off, the output growth rate $\widehat{h}(i^*)$ is lower (or higher) than that of the autarky case studied in Ju, Lin and Wang (2015) (*i.e.*, $\alpha = 1$ for country 1, or $\beta = 1$ for country 2) when $\sigma < 1$ (or $\sigma > 1$). The output growth rate changes again at time $t_{i^{**},0}$ when the trade partner also takes off. For example, when $i^* = 1$, the output growth rate of country 1 jumps up at $t_{2,0}$ if and only if $(\sigma - 1)\left[\alpha(1-\sigma)\xi_1 + (\beta + \alpha\sigma)\xi_2 - \rho\right] > 0$ and the growth rate would not change if $\sigma = 1$, independent of ξ_1 and ξ_2. After both countries take off ($t \geq \max\{t_{1,0}, t_{2,0}\}$), the output growth rate of country 1 exceeds that in the autarky case if and only if $(\xi_1 - \xi_2)\left(1 - \frac{1}{\sigma}\right) > 0$.

Figure 3 depicts the output path of the late bloomer, which takes off at time $t_{i^{**},0}$ and grows at a constant rate $\widetilde{h}(i^{**})$ afterwards. In particular, the growth rate in the Solow regime would be identical to that in autarky when $\sigma = 1$, independent of the ISTP rate of its trade partner ξ_{i^*}.

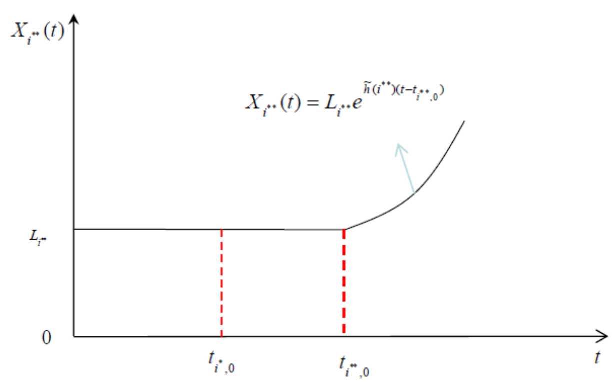

Figure 3. Time Path of Output X in Late Bloomer i^{**}

Consumption growth is given by (16), which implies that when both countries take off ($t \geq \max\{t_{1,0}, t_{2,0}\}$), the consumption grows faster when the ISTP of the trade partner is faster. Different from autarky, now a country may enjoy positive consumption growth even before it takes off due to international trade. Observe that the final consumption of the two countries always grow at the same rate, which is sensitive to the assumption that the two countries have the same Armington Cobb-Douglas aggregation in (1). If, instead, α_i is different across the two countries (due to home bias, for example), then the final consumption growth rates are generally different, but the key comparative static properties remain valid, please refer to Appendix 6.[10]

The following lemma summarizes the dynamics of the prices and terms of trade.

Lemma 3 *For any $t \geq 0$,*

$$\frac{\overset{\bullet}{P_1}(t)}{P_1(t)} = \beta \left[h_2(t) - h_1(t)\right]; \; \frac{\overset{\bullet}{P_2}(t)}{P_2(t)} = \alpha \left[h_1(t) - h_2(t)\right], \tag{19}$$

where $h_1(t)$ and $h_2(t)$ are given by Proposition 2.

Proof. Please refer to Appendix 2. ■

By revoking Proposition 2, we obtain $\frac{\overset{\bullet}{P_1}(t)}{P_1(t)} = \beta(\xi_2 - \xi_1)$ and $\frac{\overset{\bullet}{P_2}(t)}{P_2(t)} = \alpha(\xi_1 - \xi_2)$ after both countries take off. That is, the terms of trade deteriorate when a country has a faster ISTP. It is because a more efficient technology of capital good production leads to faster industrial upgrading and hence larger output, which worsens the terms of trade as the substitution elasticity between the domestic and foreign goods are positive. According to (15), $h_2 < 0$ (implying industrial downgrading) when $\xi_1 > \xi_2$ and the inter-temporal elasticity of substitution $\frac{1}{\sigma}$ is sufficiently small. Even in that case, however, country 2 still enjoys a positive consumption growth despite the negative output growth, because the terms of trade become increasingly favorable. This "immiserizing growth" result echoes the result highlighted by Acemoglu and Ventura (2002), who try to explain the stationarity of the world income distribution observed in the data.

For the rest of the paper, we will mainly focus on the case of industrial upgrading with $h_1(t)$ and $h_2(t)$ both weakly positive. Industrial downgrading can be analyzed in a similar way. To satisfy the transversality condition, we further impose

$$a < \lambda^{\frac{\xi_i}{h_i}} \tag{20}$$

for both $i = 1$ and 2. Intuitively, if the capital intensity parameter a is too large to satisfy (20), then capital accumulation is not sustainable to ensure a positive consumption and output growth. Please refer to Appendix 3 for further discussions of (20).

The following proposition highlights how output growth is affected by domestic and foreign ISTP in the long run.

[10]Notice that the share of import or export in the total GDP is endogenous, not necessarily fixed, as GDP incorporates both consumption goods and capital goods.

Proposition 4 *When* $t \geq \max\{t_{1,0}, t_{2,0}\}$*, the following is true:[1]*$\frac{\partial h_i}{\partial \xi_i} > 0$ *for any* $\sigma > 0$*,* $\forall i \in \{1,2\}$*; [2]*$\frac{\partial h_i}{\partial \xi_j} \begin{cases} > 0 & when \quad \sigma \in (0,1) \\ = 0 & when \quad \sigma = 1 \\ < 0 & when \quad \sigma \in (1,\infty) \end{cases}$*, for* $\forall i, j \in \{1,2\}$ *and* $i \neq j$*.*

Proof. Immediately implied by Proposition 2. ■

Part [1] is straightforward. Part [2] points to the importance of the inter-temporal elasticity of substitution in determining how the output growth rate is affected by the trade partner's ISTP. The output growth is mainly determined by how fast capital accumulates via endogenous saving. Suppose the ISTP rate in country 2 (ξ_2) increases, it will generate two opposite effects. The first effect is the inter-temporal substitution effect, which facilitates output growth in country 1. The reason is as follows: By revoking Lemma 3, imports to country 1 become increasingly cheaper in the future, so this dynamic change in terms of trade implies that households in country 1 should substitute today's consumption for tomorrow's consumption, or in other words, country 1 should save more today. Consequently, output grows faster due to quicker capital accumulation. The second effect is the inter-temporal income effect, which slows down the output growth for the following reason: A larger ξ_2 implies that the real income of country 1 becomes higher, partly because its import from country 2 becomes increasingly cheaper, and partly because the market size of country 2 increases faster, so country 1's export revenue also grows faster. Higher real income implies that households in country 1 should consume more (and hence save less) at each time point including today, so the output growth is dragged down due to slower capital accumulation.

It turns out that if and only if the inter-temporal elasticity of substitution $\frac{1}{\sigma}$ is strictly larger than unity, the inter-temporal substitution effects will dominate the inter-temporal income effect, so consumers will save more when the ISTP rate of the trade partner increases, which accelerates the output growth ($\frac{\partial h_1}{\partial \xi_2} > 0$ and $\frac{\partial h_2}{\partial \xi_1} > 0$). When the inter-temporal elasticity of substitution is unity, these two opposite effects exactly cancel out each other, so a country's output growth is independent of its trade partner's ISTP.

3.2 Industry Dynamics

Now we derive the industry dynamics. Since we mainly focus on the case with $h_i > 0$, $t_{i,n}$ must be strictly increasing in n for both $i = 1$ and 2. The following proposition characterizes the whole life cycle dynamics of each industry on the aggregate growth path.

Proposition 5 *Suppose* $K_i(0)$ *is sufficiently small for a given country* $i \in \{1,2\}$*. In the dynamic free-trade equilibrium, each industry has a hump-shaped life cycle. More* *precisely, industry output for any good n at time t in country i is given by the following:*

$$x^*_{i,n}(t) = \begin{cases} \frac{L_i \exp\left(\int_{t_{i,0}}^t h_i(s)ds\right)}{\lambda^n - \lambda^{n-1}} - \frac{L_i}{\lambda-1} & when \quad t \in [t_{i,n-1}, t_{i,n}] \\ -\frac{L_i \exp\left(\int_{t_{i,0}}^t h_i(s)ds\right)}{\lambda^{n+1} - \lambda^n} + \frac{\lambda L_i}{\lambda-1}, & when \quad t \in [t_{i,n}, t_{i,n+1}] \\ 0, & otherwise \end{cases}, \text{ for all } n \geq 2$$

$$x^*_{i,1}(t) = \begin{cases} \frac{L_i \exp\left(\int_{t_{i,0}}^t h_i(s)ds\right) - L_i}{\lambda-1}, & when \quad t \in [t_{i,0}, t_{i,1}] \\ -\frac{L_i \exp\left(\int_{t_{i,0}}^t h_i(s)ds\right)}{\lambda^2 - \lambda} + \frac{\lambda L_i}{\lambda-1}, & when \quad t \in [t_{i,1}, t_{i,2}] \\ 0, & otherwise \end{cases},$$

$$x^*_{i,0}(t) = \begin{cases} L_i - \frac{L_i \exp\left(\int_{t_{i,0}}^t h_i(s)ds\right) - L_i}{\lambda-1}, & when \quad t \in [t_{i,0}, t_{i,1}] \\ L_i, & when \quad t \in [0, t_{i,0}] \end{cases},$$

where $h_i(\cdot)$ is given by Proposition 2 and the critical time points $\{t_{i,n}\}_{n=0}^{\infty}$ are to be given by Proposition 8.

Proof. Using Table 1 and the fact that $X_i(t) = L_i$ for any $t \leq t_{i,0}$ and $X_i(t) = L_i \exp\left(\int_{t_{i,0}}^t h_i(s)ds\right)$ for any $t \geq t_{i,0}$. $h_i(\cdot)$ is given by Proposition 2. ■

This proposition can be illustrated more intuitively by the hump-shaped life cycle industry dynamics in Figure 4.

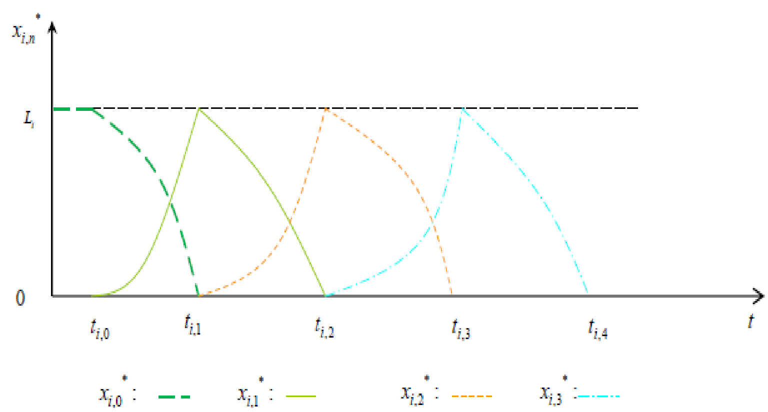

Figure 4. Industrial Dynamics in Country i with International Trade when $t_{i,0} > 0$.

The economy is initially in the Malthusian trap with stagnant output per capita till it takes off at $t_{i,0}$, after which the economy switches to the Solow regime with per capita output growth rate $h_i(t)$. Beneath this sustainable aggregate growth path, the underlying industries are upgrading endogenously along the ladder of capital intensity. Each industry follows a hump-shaped life cycle and more capital-intensive industries reach their peaks later. These dynamic patterns are qualitatively consistent with the empirical facts documented in the literature (see, *e.g.*, Chenery *et al* (1986), Schott (2003) and Haraguchi and Rezonja (2010)).

Different from the autarky economy in Ju, Lin, Wang (2015), now industry dynamics in a country is affected by its trade partner, as the output growth rates of the two countries are generally interdependent (recall Proposition 2). More formally, define $m_{i,n} \equiv t_{i,n+1} - t_{i,n}$, which measures how long goods n and $n+1$ coexist in country $i \in \{1,2\}$ for $n \geq 0$ (*i.e.*, the duration for the diversification cone containing goods n and $n+1$).

If country i takes off earlier than its trade partner (*i.e*, $i = i^*$), its output growth generically changes twice as depicted in Figure 2. Let $\widehat{n}$ denote the index of the least capital-intensive industry in the early bloomer that reaches its peak weakly after country i^{**} takes off (*i.e.*, $t_{i^*,\widehat{n}-1} < t_{i^{**},0} \leq t_{i^*,\widehat{n}}$), where $\widehat{n} \geq 1$. Alternatively speaking, when country i^{**} takes off, industry $\widehat{n}-1$ is declining and industry $\widehat{n}$ is booming. It is straightforward to show

$$m_{i^*,n} = \begin{cases} \widetilde{m}(i^*) \equiv \frac{\log\lambda}{\widetilde{h}(i^*)}, & \forall n \geq \widehat{n}+1 \\ \widehat{m}(i^*) \equiv \frac{\log\lambda}{\widehat{h}(i^*)}, & \text{when } 0 \leq n \leq \widehat{n}-2 \\ \overline{m}(i^*) & \text{when } n = \widehat{n}-1 \end{cases}, \tag{21}$$

where $\widehat{h}(\cdot)$ and $\widetilde{h}(\cdot)$ are given by (17) and (18), respectively, and

$$\overline{m}(i^*) \equiv \frac{\log\lambda}{\widetilde{h}(i^*)} + (1 - \frac{\widehat{h}(i^*)}{\widetilde{h}(i^*)})\left(t_{i^{**},0} - t_{i^*,0} - (\widehat{n}-1)\,\widehat{m}(i^*)\right), \tag{22}$$

with $t_{i^{**},0}$ and $t_{i^*,0}$ to be determined in Proposition 8. (21) indicates that it is the takeoff of the late bloomer that permanently alters the life spans of each industry.

If, on the other hand, country i takes off later (that is, $i = i^{**}$), then for $\forall t \in [t_{i^{**},0}, \infty)$, we must have $h_{i^{**}}(t) = \widetilde{h}(i^{**})$, as depicted in Figure 3. In that case, all industries have equal life spans:

$$m_{i^{**},n} = \widetilde{m}(i^{**}) \equiv \frac{\log\lambda}{\widetilde{h}(i^{**})}, \forall n \geq 0. \tag{23}$$

Based on (21) and (23), we obtain the following proposition to characterize the life span of each industry in the two countries.

Proposition 6 *The life spans of existing industries are different in the early bloomer (country i^*) before and after its trade partner takes off. In particular, each industry $n \geq 1$ that disappears before the trade partner takes off has the same life span, equal to $2\widehat{m}(i^*)$; industry $\widehat{n}-1$ has the life span equal to $\widehat{m}(i^*)+\overline{m}(i^*)$, whereas industry $\widehat{n}$ has the life span equal to $\overline{m}(i^*)+\widetilde{m}(i^*)$. In contrast, the life span of every industry $n \geq 1$in the late bloomer (country i^{**}) is equal to $2\widetilde{m}(i^{**})$. The industry upgrading speed, measured by the reciprocal of the life span, increases with the average output growth rate but decreases with λ.*

Different from autarky, now the life span of an industry depends on the characteristics of both countries via trade. Moreover, (21) and (23) suggest that the industrial upgrading speed is proportional to the output growth rate. This endogenous synchronization of individual industry dynamics and aggregate growth further implies that, by revoking Proposition 4, the industrial upgrading of a country can be either facilitated or hampered by its trade partner's ISTP, depending on whether the inter-temporal elasticity of substitution is larger or smaller than unity. Moreover, the country with faster ISTP (higher ξ) will experience more rapid industry upgrading.

When λ increases, the productivities of the two neighboring industries both increase (recall assumptions (4) and (5)), which creates two opposite effects. The productivity increase in the higher-indexed industry induces a faster upgrading whereas the productivity increase in the lower-indexed industry induces a longer stay at the current industry. The assumption $a - 1 > \lambda$ in (5) implies that the second effect dominates, thus a larger λ implies slower industrial upgrading.

Proposition 6 also indicates that the life cycles of industries in the early bloomer is permanently altered by the takeoff of its trade partner. For concreteness, suppose $i^* = 1$. Before country 2 takes off, the complete life span for each industry in country 1 is equal to $\frac{2(\beta+\alpha\sigma)\log\lambda}{\xi_1-\rho}$, independent of the ISTP of country 2. However, after country 2 takes off at $t_{2,0}$, the life span for each new industry permanently shifts to $\frac{2\log\lambda}{\beta(\xi_1-\xi_2)+\frac{\alpha\xi_1+\beta\xi_2-\rho}{\sigma}}$, so the industry upgrading in country 1 becomes faster (or slower) after country 2 takes off if and only if $(\sigma-1)\left[\alpha(1-\sigma)\xi_1+(\beta+\alpha\sigma)\xi_2-\rho\right]>0$ (or <0). For instance, when $\xi_2 \geq \xi_1$, the life span of each new industry in country 1 will be shorter (longer) after country 2 takes off, if and only if the inter-temporal elasticity of substitution $\frac{1}{\sigma}$ is smaller (larger) than one. In the special case when the inter-temporal elasticity of substitution is equal to one ($\sigma = 1$), the pace of industry upgrading is always independent of the ISTP of the trade partner and also time invariant in both countries. The intuition is essentially similar to that for Proposition 4.

Given the key role of capital accumulation in driving the economic takeoff, industry dynamics, and aggregate growth, it is natural to ask how these dynamics depend on the initial capital stocks of the two countries, which is addressed by the following proposition.

Proposition 7 *Given $K_i(0) = K_{i,0}$, country i starts by producing only good 0 if $0 < K_{i,0} \leq \vartheta_{i,0}$ and starts by producing goods n and $n+1$ if $\vartheta_{i,n} < K_{i,0} \leq \vartheta_{i,n+1}$, for any $n \geq 0$, where $\{\vartheta_{i,n}\}_{n=0}^{\infty}$ is an increasing sequence of strictly positive numbers. In* *particular, when $i = i^{**}$, we have*

$$\vartheta_{i,0} \equiv \frac{ah_i\left(1-\lambda^{1-\frac{\xi_i}{h_i}}\right)\left(1-\lambda^{\frac{\xi_i}{h_i}}\right)}{(\lambda-1)\,\lambda^{\frac{\xi_i}{h_i}}\xi_i\,(h_i-\xi_i)\left(1-a\lambda^{\frac{-\xi_i}{h_i}}\right)}L_i, \tag{24}$$

$$\vartheta_{i,n} \equiv \frac{a^n\left[\xi_i\left(a-\lambda^{\frac{\xi_i}{h_i}}\right)(\lambda-1)+h_i(a-\lambda)\left(1-\lambda^{\frac{\xi_i}{h_i}}\right)\right]}{(\lambda-1)\,\lambda^{\frac{\xi_i}{h_i}}\xi_i\,(h_i-\xi_i)\left(1-a\lambda^{\frac{-\xi_i}{h_i}}\right)}L_i, \text{ for any } n \geq 1,$$

where $h_i = \widetilde{h}(i)$ given by (18). When $i = i^*$, (24) still holds with $h_i = \widehat{h}(i)$ given by (17).

Proof. Refer to Appendix 3. ■

Observe that the threshold values for capital $\{\vartheta_{i,n}\}_{n=0}^{\infty}$ are proportional to the domestic labor endowment, so what matters is the capital-labor ratio. For the late bloomer, $\{\vartheta_{i^{**},n}\}_{n=0}^{\infty}$ also depend on the ISTP of the trade partner via $h_{i^{**}}$, but independent of the initial capital endowments of both countries to the extent that the trade partner takes off earlier. Observe that $\xi_i > \widetilde{h}(i)$ due to (12). It can be shown that $\frac{\partial\vartheta_{i,0}}{\partial h_i} > 0$, which, by revoking Proposition 4, implies that $\frac{\partial\vartheta_{i^{**},0}}{\partial\xi_{i^*}} \gtreqless 0$ if $\sigma \lesseqgtr 1$. Moreover, $\frac{\partial\vartheta_{i^{**},0}}{\partial\xi_{i^{**}}} > 0$ if $\sigma \geq 1$. In other words, domestic and foreign ISTP have opposite impacts on the threshold value of capital for economic takeoff in the late bloomer when the inter-temporal elasticity of substitution is no larger than unity.

Proposition 8 *Suppose $K_{i,0} \in (0, \vartheta_{i,0}]$ in some country $i \in \{1,2\}$. In the dynamic free-trade equilibrium, the time of taking off in country i is given by*

$$t_{i,0} = \frac{1}{\xi_i}\left[\log\frac{ah_i\left(1-\lambda^{1-\frac{\xi_i}{h_i}}\right)\left(1-\lambda^{\frac{\xi_i}{h_i}}\right)}{(\lambda-1)\,\lambda^{\frac{\xi_i}{h_i}}\xi_i\,(h_i-\xi_i)\left(1-a\lambda^{\frac{-\xi_i}{h_i}}\right)} - \log\frac{K_{i,0}}{L_i}\right], \tag{25}$$

where $h_i = \widehat{h}(i)$ given by (17) if $i = i^*$ and $h_i = \widetilde{h}(i)$ given by (18) if $i = i^{**}$. For any $n \geq 1$, we have

$$t_{i,n} = t_{i,0} + \sum_{n'=0}^{n-1} m_{i,n'}, \tag{26}$$

where $m_{i,n'}$ is given by (21) or (23) depending on whether $i = i^*$ or i^{**}.

Proof. Observe that

$$t_{i,0} = \frac{\log\frac{\vartheta_{i,0}}{K_{i,0}}}{\xi_i}. \tag{27}$$

where $\vartheta_{i,0}$ is given by (24). (26) is straightforward. ■

(25) implies that a country with a smaller initial capital-labor ratio takes off later ($\frac{\partial t_{i,0}}{\partial\left(\frac{K_{i,0}}{L_i}\right)} < 0$) and a larger is later if the is smaller or if the capital requirement parameter a is larger. In addition, for the early bloomer i^*, the foreign ISTP does not affect its time to take off ($\frac{\partial t_{i^*,0}}{\partial \xi_{i^{**}}} = 0$). However, the late bloomer i^{**}, $\frac{\partial t_{i^{**},0}}{\partial \xi_{i^*}} \gtreqless 0$ if $\sigma \lesseqgtr 1$. The intuition is almost identical to that for Proposition 4: When the foreign rate of ISTP increases, the inter-temporal terms-of-trade effect (substitution effect) tends to delay the take-off in order to boost saving and facilitate capital accumulation, whereas the market-size effect (income effect) tends to reduce saving and expedite the economic take-off. The first effect dominates if and only if the inter-temporal elasticity of substitution exceeds one ($0 < \sigma < 1$).

It remains to characterize the time path of capital stock $K_i(t)$. We focus on the late bloomer $i = i^{**}$. Define, for any $n \geq 1$,

$$\alpha_{i,n} = -\frac{a^n (a-\lambda) L_i}{\xi_i (\lambda - 1)}, \tag{28}$$

$$\beta_{i,n} = -\left(\frac{a^{n+1} - a^n}{\lambda^{n+1} - \lambda^n}\right) \frac{X_i(0)}{(h_i - \xi_i)}, \tag{29}$$

$$\gamma_{i,n} = \left[\frac{\lambda^n L_i}{X_i(0)}\right]^{\frac{-\xi_i}{h_i}} \left\{\vartheta_{i,n} + \frac{(a^{n+1} - a^n) L_i}{\lambda - 1}\left[\frac{1}{(h_i - \xi_i)} + \frac{(a-\lambda)}{\xi_i (a-1)}\right]\right\}. \tag{30}$$

Proposition 9 *Suppose $K_{i,0} \in (0, \vartheta_{i,0}]$.*[11] *The initial output of country i is $X_i(0) = L_i$,and the capital accumulation function is*

$$K_i(t) = \begin{cases} K_{i,0} e^{\xi_i t}, & \text{for } t \in [0, t_{i,0}] \\ \frac{-\frac{aL_i}{\lambda-1}}{h_i - \xi_i} e^{h_i t} + \frac{-aL_i}{\xi_i(\lambda-1)} + \left[\vartheta_{i,0} + \frac{\frac{aL_i}{\lambda-1}}{h_i-\xi_i} + \frac{aL_i}{\xi_i(\lambda-1)}\right] e^{\xi_i t} & \text{for } t \in [t_{i,0}, t_{i,1}] \\ \alpha_{i,n} + \beta_{i,n} e^{h_i t} + \gamma_{i,n} e^{\xi_i t}, & \text{for } t \in [t_{i,n}, t_{i,n+1}], \text{ any } n \geq 1 \end{cases};$$

where $\alpha_i, n, \beta_i, n,$and γ_i, n are defined by (28)-(30), respectively, t_i, n is given by (25) or (26); and $\vartheta_i, 0$ is given by (24), $h_i = \widetilde{h}(i)$ given by (18) for any country $i = i^* * \in \{1, 2\}$ and any $n \geq 0$.

Proof. See Appendix 3. ■

The functional form of the capital evolution equation is changing over time, reflecting the structural change (industrialization) and the composition shifts of the underlying industries. Once $X_1^*(0)$ and $X_2^*(0)$ are determined, by revoking (8), we obtain the initial aggregate consumptions as follows

$$C_1^*(0) = \alpha X_1^{*\alpha}(0) X_2^{*\beta}(0); \; C_2^*(0) = \beta X_1^{*\alpha}(0) X_2^{*\beta}(0).$$

[11] In Appendix 3, we also characterize the cases when this condition does not hold.

Since $X_i(t) = L_i$ for any $t \leq t_{i,0}$ and $X_i(t) = L_i e^{\int_{t_{i,0}}^{t} h_i(s)ds}$ for any $t \geq t_{i,0}$, where $h_i(\cdot)$ is given by Proposition 2, the consumption path can be uniquely pinned down:

$$C_1^*(t) = \alpha X_1^{*\alpha}(t) X_2^{*\beta}(t);\; C_2^*(t) = \beta X_1^{*\alpha}(t) X_2^{*\beta}(t).$$

This completes the characterization of the free-trade dynamic equilibrium. Observe that $\vartheta_{i,n} \equiv K_i(t_{i,n})$ for any i and any n as long as $t_{i,n} > 0$. Different initial capital endowments only translate into different levels of the initial aggregate consumption and initial industrial composition, but they cannot affect the speed of consumption and output growth after both countries take off.

To summarize, different from autarky in Ju, Lin and Wang (2015), now the take-off time for industrialization $(t_{i,0})$, life cycle dynamics of each industry $(x_{i,n}(t))$, speed of industrial upgrading $(\frac{1}{m_i})$, output growth rate $(h_i(t))$, consumption growth rate $(\theta_i(t))$, and the capital accumulation process $(K_i(t))$ of a country are all affected by its trade partner. Generally speaking, it matters whether the inter-temporal elasticity of substitution exceeds unity or not, as it determines the sign of the impact of the trade partner's ISTP rate on almost all the aforementioned dynamics. The initial capital-labor ratios of the two countries would affect which takes off earlier and also have a level effect on output and consumption, but they have no long-lasting speed effect.

Pareto optimality is achieved because the First Welfare Theorem applies. However, nothing ensures output convergence between the two trading countries. In particular, convergence occurs in the long run if and only if the less developed country has a faster ISTP than its trade partner. In other words, free trade does not necessarily speed up the industrial upgrading.

Naturally, one may ask what happens if there exist trade barriers, which is addressed next.

4 Trade Policy and Industrial Upgrading

4.1 Static Trade Policy

Suppose country 1 imposes tariff τ_2 on the imported good from country 2 and all the tariff revenue T_1 is given to the domestic households as a lump-sum transfer. Similarly, country 2 imposes tariff τ_1 on the imported good from country 1 and all the tariff revenue T_2 is also transferred to the domestic households in a lump-sum fashion. The equilibrium is characterized in the following lemma.

Lemma 10 *In the static trade equilibrium, the total consumptions are given by*

$$C_1(\tau_1, \tau_2) = C_{1,1}^{\alpha} C_{1,2}^{\beta} = \alpha \left[\frac{(1+\tau_2)}{(1+\alpha\tau_2)} \right]^{\alpha} \left[\frac{1}{(1+\beta\tau_1)} \right]^{\beta} X_1^{\alpha} X_2^{\beta}, \tag{31}$$

and

$$C_2(\tau_1, \tau_2) = C_{2,1}^{\alpha} C_{2,2}^{\beta} = \beta \left[\frac{1}{(1 + \alpha \tau_2)} \right]^{\alpha} \left[\frac{(\tau_1 + 1)}{1 + \beta \tau_1} \right]^{\beta} X_1^{\alpha} X_2^{\beta}, \tag{32}$$

while the equilibrium terms of trade are given by

$$\frac{P_1}{P_2} = \frac{\alpha(1 + \alpha\tau_2)X_2}{\beta(1 + \beta\tau_1)X_1}. \tag{33}$$

where X_1 and X_2 are provided in Table 1.

Proof. See Appendix 4. ■

(31) and (32) indicate that the total consumption of a country increases with the tariff rate on the imported good but decreases with the tariff on its export imposed by its trade partner. This is due to the endogenous terms-of-trade effect shown in equation (33): A fixed expenditure share on imports implies that the after-tariff price of the imported good must increase relative to the export price when the tariff rate increases, because output X_1 and X_2 are fixed. Moreover, the consumption ratio of the two countries is given by

$$\frac{C_1}{C_2} = \frac{\alpha(1 + \tau_2)^{\alpha}}{\beta\, (1 + \tau_1)^{\beta}}, \tag{34}$$

which is independent of the total output. It implies that the protectionist trade policy favors domestic consumption in the world consumption distribution.

In the model, the supply side is immune from this particular type of trade policies because the tariff is imposed on the aggregate good instead of specific industries, therefore neither the marginal rate of transformation nor the equilibrium relative prices across different industries is altered by the industry-neutral trade policies within the same country. Profit-maximization of all the competitive firms plus the factor market clearing conditions leads to the same quantity of output as in a free-trade static economy. This trade policy changes the relative output prices across different countries but does not alter the relative output prices across different industries within a country.

4.2 Dynamic Trade Policy

Imagine the gross tariff rates behave as follows:

$$\frac{\dot{\tau}_1(t)}{\tau_1(t) + 1} = \phi_1(t), \frac{\dot{\tau}_2(t)}{\tau_2(t) + 1} = \phi_2(t), \tag{35}$$

where $\phi_1(t)$ and $\phi_2(t)$ are exogenous and arbitrary. The following lemma characterizes how dynamic trade policies affect the consumption growth and output growth after both countries take off.

Lemma 11 *For any exogenous dynamic trade policies specified as (35), the consumption and output growth rates for the two countries are given by*

$$\theta_1(t) = \frac{\alpha\xi_1 + \beta\xi_2 - \rho}{\sigma} - \frac{\beta}{\sigma}\left\{\alpha\phi_2(t)\left[\frac{\alpha\tau_2(t)}{1+\alpha\tau_2(t)} - \sigma\right] + \beta\phi_1(t)\left[\sigma + \frac{\alpha\tau_1(t)}{1+\beta\tau_1(t)}\right]\right\}, \tag{36}$$

$$h_1(t) = \frac{\alpha\xi_1 + \beta\xi_2 - \rho}{\sigma} + \beta(\xi_1 - \xi_2) + \beta\left\{\begin{array}{c} \phi_2(t)\alpha\left[\frac{\tau_2(t)\alpha\left[2-\frac{1}{\sigma}-\sigma\right]+1-\sigma}{1+\alpha\tau_2(t)}\right] \\ -\phi_1(t)\left[\frac{(\alpha+\beta\sigma)\left[\frac{\beta\tau_1(t)}{\sigma}-\beta\tau_1(t)-1\right]+1}{1+\beta\tau_1(t)}\right]\end{array}\right\}, \tag{37}$$

$$\theta_2(t) = \frac{\alpha\xi_1 + \beta\xi_2 - \rho}{\sigma} - \frac{\alpha}{\sigma}\left[\alpha\phi_2(t)\left(\frac{\beta\tau_2(t)}{1+\alpha\tau_2(t)} + \sigma\right) + \beta\phi_1(t)\left(\frac{\beta\tau_1(t)}{1+\beta\tau_1(t)} - \sigma\right)\right], \tag{38}$$

$$h_2(t) = \frac{\alpha\xi_1 + \beta\xi_2 - \rho}{\sigma} + \alpha(\xi_2 - \xi_1) + \alpha\left\{\begin{array}{c} \phi_1(t)\beta\left[\frac{\tau_1(t)\beta\left[2-\frac{1}{\sigma}-\sigma\right]+1-\sigma}{1+\beta\tau_1(t)}\right] \\ -\phi_2(t)\left[\frac{(\beta+\alpha\sigma)\left[\frac{\alpha\tau_2(t)}{\sigma}-\alpha\tau_2(t)-1\right]+1}{1+\alpha\tau_2(t)}\right]\end{array}\right\}. \tag{39}$$

Proof. See Appendix 5. ■

Compared with free trade (Proposition 2), the net effect of dynamic trade policies is summarized in the last term of each of the above four expressions. The following remark (an immediate corollary of Lemma 11) characterizes how trade policies affect the convergence and divergence between the trade partners.

Remark 12 *The differences in consumption growth and output growth across the two countries at any time t are given by*

$$\theta_1(t) - \theta_2(t) = \alpha\phi_2(t) - \beta\phi_1(t), \tag{40}$$

$$h_1(t) - h_2(t) = (\xi_1 - \xi_2) + (\alpha\phi_2(t) - \beta\phi_1(t))(1-\sigma). \tag{41}$$

In fact, (40) can be directly derived from (34) and (35). It says that whether the consumption levels of the two countries converge or diverge are fully determined by the weighted differences in the tariff change rates. Accelerating trade protection would help increase home country's speed of consumption growth relative to the trade partner. When $\alpha\phi_2(t) = \beta\phi_1(t)$, the consumption growth rates are equalized, same as in the case of free trade. (41) indicates that the inter-temporal elasticity of substitution affects the impact of dynamic trade policies on the differences in the output growth rates for the two countries. More precisely, dynamic trade policies have no impact when $\sigma = 1$. Acceleration of trade protection helps enhance the output growth advantage of home country when $\sigma \in (0, 1)$, whereas the opposite is true when $\sigma \in (1, \infty)$. When $\alpha\phi_2(t) = \beta\phi_1(t)$, the difference in output growth rates is equal to the difference in the the rates of ISTP, same as in the case of free trade. Again, the effect on the speed of industry upgrading is the same as that of output growth due to the endogenous synchronization.

Remark 13 *Any time-invariant tariff rate (i.e., $\phi_1(t) = \phi_2(t) = 0$, $\forall t$) has no effect on the long-run equilibrium growth rates and industrial upgrading speeds of each country.*

The absence of growth effect results from the fact that time-invariant tariffs do not distort the production activities within each country, because tariffs can only distort the terms of trade, but not the marginal rate of transformation within each country. Consequently, tariffs cause deadweight loss only in terms of consumption and welfare, as indicated by (31) and (32), but not in production. When tariff rates vary over time, consumption and output growth rates may change. Consider the long-run growth effect of gradual trade liberalization in country 1 (that is, $\phi_2(t) \leq 0$), which is summarized in the following proposition, another immediate implication of Lemma 11.

Proposition 14 *After both countries take off, trade liberalization of home country affects its own growth in the following way: [1] When* $\sigma \in (0,1)$, $\frac{\partial \theta_1(t)}{\partial |\phi_2(t)|} \begin{cases} > 0, & when \quad \tau_2(t) > \tau_2^* \\ = 0, & when \quad \tau_2(t) = \tau_2^* \\ < 0, & when \quad \tau_2(t) < \tau_2^* \end{cases}$, $\frac{\partial h_1(t)}{\partial |\phi_2(t)|} \begin{cases} > 0, & when \quad \tau_2(t) > \tau_2^{**} \\ = 0, & when \quad \tau_2(t) = \tau_2^{**} \\ < 0, & when \quad \tau_2(t) < \tau_2^{**} \end{cases}$ *, where* $\tau_2^* \equiv \frac{\sigma}{\alpha(1-\sigma)}$ *and* $\tau_2^{**} \equiv \frac{\beta(1-\sigma)}{\frac{\alpha\beta}{\sigma} + \alpha\sigma - 2\alpha\beta}$. *[2] When* $\sigma \in [1,\infty)$, $\frac{\partial \theta_1(t)}{\partial |\phi_2(t)|} < 0$ *and* $\frac{\partial h_1(t)}{\partial |\phi_2(t)|} \geq 0$*,whenever* $\tau_2(t) \geq 0$, *"=" holds only when* $\sigma = 1$.

The first part of this proposition states that, when the inter-temporal elasticity of substitution is larger than unity ($\sigma \in (0,1)$), the consumption growth rate first increases with the speed of trade liberalization until τ_2 reaches the value τ_2^*, after which the consumption growth rate strictly decreases with the speed of trade liberalization (when $\tau_2 < \tau_2^*$). Similarly, the output growth rate also first increases when trade liberalization accelerates, but then declines with the speed of trade liberalization once the tariff rate is below τ_2^{**}. Observe that $\tau_2^{**} < \tau_2^*$, implying that the consumption growth starts to decline earlier than the output growth if trade liberalization accelerates.

The intuition for the non-monotonic impact is the following. As the tariff rate increasingly declines over time, imports become increasingly cheaper, therefore the inter-temporal substitution effect causes consumers to substitute today's consumption of imports for tomorrow through saving, which in turn increases the consumption growth rates. On the other hand, the real income becomes increasingly larger as the import price becomes increasingly cheaper. This positive income effect tends to increase the consumption and decreases the saving, which in turn tends to lower the consumption growth rate. When the tariff rate is sufficiently high, the substitution effect dominates the income effect, therefore accelerating trade liberalization increases the growth rate of consumption. However, the substitution effect becomes increasingly weaker as the tariff level goes down. Eventually, when the tariff rate is sufficiently small, the income effect dominates the substitution effect, so the growth rate of consumption starts to decline.

Symmetrically, if trade liberalization decelerates in country 1 (that is, $|\phi_2(t)|$ decreases over time), then the consumption growth rate will first decline and then increase.

To understand the impact on the output growth, first note that there are two competing effects when the tariff rate on imports decreases. One is the substitution effect which tends to decrease the domestic demand for domestic output. The second effect is the positive income effect due to the rise of real income as a result of tariff reduction. The income effect tends to increase the demand for domestic output. The net impact on the domestic output is positive because the substitution elasticity between imports and outputs is unity (Cobb-Douglas function).[12] The competitive market force dictates that output of domestic goods will increase, only partly offsetting the decline of the relative price of imports because of the balanced trade constraint (33). Consequently, an acceleration in the trade liberalization leads to an increase in the output growth rate ($\frac{\partial h_1(t)}{\partial|\phi_2(t)|} > 0$). Since only a fraction of total output is used for domestic consumption, the impact on consumption growth is smaller than that on the output growth. This is why consumption growth rate declines earlier than the output growth ($\tau_2^{**} < \tau_2^{*}$).

Part [2] of the proposition states that the non-monotonicity result disappears when the inter-temporal elasticity is smaller than one ($\sigma \in (1, \infty)$): Accelerating trade liberalization will strictly decrease the domestic consumption growth rate, because the inter-temporal substitution effect is always dominated by the inter-temporal income effect. On the other hand, accelerating trade liberalization will strictly increase the output growth rate mainly due to the increase in the external demand from the trade partner, which benefits from the trade liberalization and grows faster.

The following proposition, also implied by Lemma 11, summarizes how growth rates are affected by the trade liberalization of the trade partner in the long run.

Proposition 15 *After both countries take off, foreign trade liberalization affects the growth of home country in the following way: For any $\sigma \in (0, \infty)$, we have $\frac{\partial \theta_i(t)}{\partial|\phi_i(t)|} > 0$ and $\frac{\partial^2 \theta_i(t)}{\partial|\phi_i(t)|\partial t} < 0$ for $i = 1, 2$. In addition,* $\frac{\partial h_i(t)}{\partial|\phi_i(t)|} \begin{cases} > 0, & \textit{when} \quad \sigma \in (0,1) \\ = 0, & \textit{when} \quad \sigma = 1 \\ < 0, & \textit{when} \quad \sigma \in (1,\infty) \end{cases}.$

It states that the consumption growth rate always increases ($\frac{\partial \theta_i(t)}{\partial|\phi_i(t)|} > 0$) when the trade partner unilaterally accelerates trade liberalization, but the marginal impact is diminishing over time ($\frac{\partial^2 \theta_i(t)}{\partial|\phi_i(t)|\partial t} < 0$) due to the tariff reduction. On the other hand, accelerating trade liberalization by the trade partner may increase or decrease the output growth rate of the home country, depending on whether the inter-temporal elasticity of

[12]In the appendix, we explore a more general model where tariff affects the expenditure share of imports. It is shown that a unilateral tariff reduction may increase or decrease the growth rates of consumption and output, depending on (1) whether the intertemporal elasticity of substitution is larger than unity; (2) whether the home country has a higher ISTC rate, and (3) whether the marginal change in the expenditure share on imports is sufficiently sensitive to a tariff reduction in the foreign country.

substitution is larger or smaller than one. The intuition can be understood in a similar way as before.

The previous two propositions suggest that it is important to go beyond static trade models to understand the growth effects of trade liberalization. The speed of trade liberalization matters! The same policy change may affect the output or consumption growth in the opposite directions, depending on the current tariff level and/or whether the inter-temporal elasticity of substitution is larger than unity.

5 Conclusion

A highly tractable two-country growth model with trade is developed to study how international trade and dynamic trade policies affect industrialization, life cycle industry dynamics, and aggregate growth. We obtain closed-form solutions to fully characterize the hump-shaped life cycle dynamics of each underlying industries beneath the sustained aggregate growth path. It is shown that the trade impact is generally different for the early bloomer and the late bloomer. We also find that the inter-temporal elasticity of substitution is a crucial parameter, which often determines the sign of the impact of the ISTP of the trade partner and that of the dynamic trade policies. This is because it determines whether the inter-temporal terms-of-trade (substitution) effect dominates the dynamic market-size (income) effect as these two effects have opposite impact on the endogenous saving decisions. The former dominates the latter when the elasticity exceeds one and the two effects exactly cancel out when the elasticity equals one. This elasticity also affects the growth impact of trade liberalization. In particular, when the elasticity exceeds one, accelerating trade liberalization first increases the rates of consumption and output growth as well as the speed of industry upgrading in the home country till the tariff rate becomes sufficiently small, after which the effect becomes exactly the opposite.

There are several directions for future research. One is to return to the traditional multiple-cone HO framework by relaxing the Armington assumption, a dynamic version of Dornbusch, Fischer and Samuelson (1980). Whereas conceptually this approach appears natural and attractive to explore life-cycle industry dynamics, the key challenge is low analytic tractability. Not only would it enormously increase the degree of nonlinearity (as industries within a country must be assumed imperfectly substitable now), but also it would exacerbate the curse of dimensionality. Recall this high-dimensionality problem is bypassed in the current model due to the endogenous exit of industries because of the linearity assumption. Pursuing this direction would presumably involve intensive quantitative characterization with numerical methods. A second direction is to allow for unbalanced trade and capital flow (see Costinot, Lorenzoni and Werning (2013) and Keyu Jin (2012)). Another direction is to introduce the productivity heterogeneity of firms into each industry by extending Bernard, Redding and Schott (2007), which may shed new light on the firm dynamics in the process of industry upgrading.

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Appendix

Appendix 1. Proof of Proposition 2. To solve the above dynamic problem, we set the *discounted-value* Hamiltonian in the interval of $t_{1,n} \leq t \leq t_{1,n+1}$ for any $n \geq 1$, and use subscripts "$n, n+1$" to denote all variables in this interval:

$$H_{n,n+1} = \frac{C_1(t)^{1-\sigma}-1}{1-\sigma}e^{-\rho t} + \eta_{n,n+1}\left[\xi_1 K_1(t) - \left[\left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)}\right]^{\frac{1}{\alpha}} - \frac{\lambda^n(a-\lambda)}{a-1}L_1\right]\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\right]$$
$$+ \zeta_{n,n+1}^{n+1}(\lambda^{n+1}L_1 - \left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)}\right]^{\frac{1}{\alpha}}) + \zeta_{n,n+1}^{n}(\left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)}\right]^{\frac{1}{\alpha}} - \lambda^n L_1) \quad (42)$$

where $\eta_{n,n+1}$ is the co-state variable, $\zeta_{n,n+1}^{n+1}$ and $\zeta_{n,n+1}^{n}$ are the Lagrangian multipliers for the two constraints $\lambda^{n+1}L_1 - C_1(t) \geq 0$ and $C_1(t) - \lambda^n L_1 \geq 0$, respectively. The first order and K-T conditions are

$$\frac{\partial H_{n,n+1}}{\partial C_1} = C_1(t)^{-\sigma}e^{-\rho t} - \left(\eta_{n,n+1}\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n} + \zeta_{n,n+1}^{n+1} - \zeta_{n,n+1}^{n}\right)\frac{1}{\alpha^2 X_2^{\beta}(t)}\left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)}\right]^{\frac{1}{\alpha}-1} = 0 \quad (43)$$

$$\zeta_{n,n+1}^{n+1}(\lambda^{n+1}L_1 - \left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)}\right]^{\frac{1}{\alpha}}) = 0;\ \zeta_{n,n+1}^{n+1} \geq 0,\ \lambda^{n+1}L_1 - \left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)}\right]^{\frac{1}{\alpha}} \geq 0$$

$$\zeta_{n,n+1}^{n}(\left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)}\right]^{\frac{1}{\alpha}} - \lambda^n L_1) = 0,\ \ \zeta_{n,n+1}^{n} \geq 0,\ \left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)}\right]^{\frac{1}{\alpha}} - \lambda^n L_1 \geq 0.$$

We also have

$$\eta'_{n,n+1}(t) = -\frac{\partial H_{n,n+1}}{\partial K_1} = -\eta_{n,n+1}\xi_1. \quad (44)$$

In particular, when $\left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)}\right]^{\frac{1}{\alpha}} \in (\lambda^n L_1, \lambda^{n+1}L_1)$, $\zeta_{n,n+1}^{n+1} = \zeta_{n,n+1}^{n} = 0$, and equation (43) becomes

$$C_1(t)^{-\sigma}e^{-\rho t} = \eta_{n,n+1}\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\frac{1}{\alpha^2 X_2^{\beta}(t)}\left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)}\right]^{\frac{1}{\alpha}-1}. \quad (45)$$

The left hand side is the marginal utility gain by increasing one unit of aggregate consumption, while the right hand side is the marginal utility loss due to the decrease in capital because of that additional unit of consumption, which by chain's rule can be decomposed into three multiplicative terms: the marginal utility of capital $\eta_{n,n+1}$, the marginal capital requirement for each additional unit of aggregate consumption $\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}$ and the terms of trade $\frac{1}{\alpha^2 X_2^{\beta}(t)}\left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)}\right]^{\frac{1}{\alpha}-1}$. Taking log of both sides of equation (45) and differentiating with respect to t, we have:

$$(\frac{1}{\alpha} + \sigma - 1)\frac{\dot{C_1}(t)}{C_1(t)} = \xi_1 - \rho + \frac{\beta}{\alpha}\frac{\dot{X_2}(t)}{X_2(t)}, \quad (46)$$

for $t_{1,n} \leq t \leq t_{1,n+1}$ for any $n \geq 0$.

Symmetrically, we also have

$$(\frac{1}{\beta}+\sigma-1)\frac{\dot{C_2}(t)}{C_2(t)}=\xi_2-\rho+\frac{\alpha}{\beta}\frac{\dot{X_1}(t)}{X_1(t)}. \tag{47}$$

Recall we have (8) hold for any time, which implies

$$\frac{\dot{C_1}(t)}{C_1(t)}=\frac{\dot{C_2}(t)}{C_2(t)}=\alpha\frac{\dot{X_1}(t)}{X_1(t)}+\beta\frac{\dot{X_2}(t)}{X_2(t)}. \tag{48}$$

Therefore we obtain (16). For the completeness of the proof, observe that the strictly concave utility function implies that the optimal consumption flow $C_1(t)$ must be continuous and sufficiently smooth (with no kinks) throughout the time, hence from (48) we obtain:

$$C_1(t)=C_1(t_{1,0})e^{\frac{\alpha\xi_1+\beta\xi_2-\rho}{\sigma}(t-t_{1,0})} \text{ for any } t\geq t_{1,0}. \tag{49}$$

Following Kamien and Schwartz (1991), we have two additional necessary conditions at $t=t_{1,n+1}$:

$$H_{n,n+1}(t_{1,n+1})=H_{n+1,n+2}(t_{1,n+1}) \tag{50}$$

$$\eta_{n,n+1}(t_{1,n+1})=\eta_{n+1,n+2}(t_{1,n+1}) \tag{51}$$

Substituting equations (50) and (51) into (42), we can verify that $K_1^-(t_{1,n+1})=K_1^+(t_{1,n+1})$. In other words, $K_1(t)$ is indeed continuous.

When $t\leq t_{1,0}$,

$$H_0=\frac{C_1(t)^{1-\sigma}-1}{1-\sigma}e^{-\rho t}+\eta_0\xi_1K_1(t)+\zeta_0(L_1-\left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}}) \tag{52}$$

FOCs and K-T conditions:

$$\begin{aligned}
C_1(t)^{-\sigma}e^{-\rho t} &= \frac{1}{\alpha}\zeta_0\left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}-1}\frac{1}{\alpha X_2^\beta(t)}\\
\eta_0'(t) &= -\frac{\partial H_{n,n+1}}{\partial K_1}=-\eta_0\xi_1.\\
\zeta_0\geq 0, L_1-&\left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}}\geq 0 \text{ and } \zeta_0(L_1-\left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}})=0.
\end{aligned}$$

therefore, we have $\frac{\dot{X_1}(t)}{X_1(t)}=0$, $\frac{\dot{C_1}(t)}{C_1(t)}=\beta\frac{\dot{X_2}(t)}{X_2(t)}$. And $\frac{\dot{\zeta_0}(t)}{\zeta_0(t)}=\rho-\beta(1-\sigma)\frac{\dot{X_2}(t)}{X_2(t)}$.

When $t \in (t_{1,0}, t_{1,1})$,

$$H_{0,1} = \frac{C_1(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} + \eta_{0,1} \left[\xi_1 K_1(t) - \frac{a}{\lambda - 1} \left[\left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)} \right]^{\frac{1}{\alpha}} - L_1 \right] \right]$$
$$+ \zeta_{0,1}^1 (\lambda L_1 - \left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)} \right]^{\frac{1}{\alpha}}) + \zeta_{0,1}^0 (\left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)} \right]^{\frac{1}{\alpha}} - L_1) \tag{53}$$

Optimality conditions state that

$$C_1(t)^{-\sigma} e^{-\rho t} - \left(\eta_{0,1} \frac{a}{\lambda - 1} + \zeta_{0,1}^1 - \zeta_{0,1}^0 \right) \frac{1}{\alpha^2 X_2^{\beta}(t)} \left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)} \right]^{\frac{1}{\alpha} - 1} = 0 \tag{54}$$

$$\zeta_{0,1}^1 (\lambda L_1 - \left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)} \right]^{\frac{1}{\alpha}}) = 0; \; \zeta_{0,1}^1 \geq 0, \lambda L_1 - \left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)} \right]^{\frac{1}{\alpha}} \geq 0$$

$$\zeta_{0,1}^0 (\left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)} \right]^{\frac{1}{\alpha}} - L_1) = 0, \; \zeta_{0,1}^0 \geq 0, \left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)} \right]^{\frac{1}{\alpha}} - L_1 \geq 0.$$

We also have

$$\eta_{0,1}'(t) = -\frac{\partial H_{0,1}}{\partial K_1} = -\eta_{0,1} \xi_1. \tag{55}$$

thus we have

$$C_1(t)^{-\sigma} e^{-\rho t} = \eta_{0,1} \frac{1}{\lambda - 1} \frac{1}{\alpha X_2^{\beta}(t)} \left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)} \right]^{\frac{\beta}{\alpha}}$$

implying

$$(\frac{1}{\alpha} + \sigma - 1) \frac{\dot{C_1}(t)}{C_1(t)} = \xi_1 - \rho + \frac{\beta}{\alpha} \frac{\dot{X_2}(t)}{X_2(t)}.$$

at the same time

$$X_1(t) = \left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)} \right]^{\frac{1}{\alpha}}$$

If $t < t_{2,0}$ holds, $\frac{\dot{X_2}(t)}{X_2(t)} = 0$ and $\frac{\dot{C_2}(t)}{C_2(t)} = \alpha \frac{\dot{X_1}(t)}{X_1(t)}$. If $t > t_{2,0}$ holds, $\frac{\dot{X_2}(t)}{X_2(t)} = 0$ and $\frac{\dot{C_2}(t)}{C_2(t)} = \alpha \frac{\dot{X_1}(t)}{X_1(t)}$

Consequently, when $t < \min\{t_{1,0}, t_{2,0}\}$, we must have $X_1(t) = L_1; X_2(t) = L_2$; $C_1(t) = \alpha L_1^{\alpha} L_2^{\beta}$; $C_2(t) = \beta L_1^{\alpha} L_2^{\beta}$. In other words,

$$\frac{\dot{C_1}(t)}{C_1(t)} = \frac{\dot{C_2}(t)}{C_2(t)} = \frac{\dot{X_1}(t)}{X_1(t)} = \frac{\dot{X_2}(t)}{X_2(t)} = 0.$$

Suppose, $t_{1,0} \neq t_{2,0}$, then when $t \in [t_{1,0}, t_{2,0}]$, we have

$$(\frac{1}{\alpha} + \sigma - 1)\frac{\dot{C_1}(t)}{C_1(t)} = \xi_1 - \rho + \frac{\beta}{\alpha}\frac{\dot{X_2}(t)}{X_2(t)}.$$

at the same time,

$$\frac{\dot{X_2}(t)}{X_2(t)} = 0, \frac{\dot{C_2}(t)}{C_2(t)} = \alpha\frac{\dot{X_1}(t)}{X_1(t)}, \frac{\dot{C_2}(t)}{C_2(t)} = \frac{\dot{C_1}(t)}{C_1(t)}$$

thus

$$\frac{\dot{C_1}(t)}{C_1(t)} = \frac{\dot{C_2}(t)}{C_2(t)} = \frac{\xi_1 - \rho}{\frac{\beta}{\alpha} + \sigma}$$

$$\frac{\dot{X_1}(t)}{X_1(t)} = \frac{\xi_1 - \rho}{\beta + \alpha\sigma}; \frac{\dot{X_2}(t)}{X_2(t)} = 0.$$

Symmetrically, when $t \in [t_{2,0}, t_{1,0}]$, then

$$\frac{\dot{C_1}(t)}{C_1(t)} = \frac{\dot{C_2}(t)}{C_2(t)} = \frac{\xi_2 - \rho}{\frac{\alpha}{\beta} + \sigma}$$

$$\frac{\dot{X_2}(t)}{X_2(t)} = \frac{\xi_2 - \rho}{\alpha + \beta\sigma}; \frac{\dot{X_1}(t)}{X_1(t)} = 0.$$

Q.E.D.

Appendix 2: Proof. of Lemma 3:First notice (13) implies

$$\frac{\dot{P_1}(t)}{P_1(t)} - \frac{\dot{P_2}(t)}{P_2(t)} = \frac{\dot{X_2}(t)}{X_2(t)} - \frac{\dot{X_1}(t)}{X_1(t)} = \xi_2 - \xi_1. \tag{56}$$

In addition, recall the price for the final good is normalized to unity at any time point, that is,

$$\left(\frac{P_1(t)}{\alpha}\right)^{\alpha}\left(\frac{P_2(t)}{\beta}\right)^{\beta} = 1,$$

which implies

$$\alpha\frac{\dot{P_1}(t)}{P_1(t)} + \beta\frac{\dot{P_2}(t)}{P_2(t)} = 0. \tag{57}$$

(56) and (57) jointly yield (19). **Q.E.D.**

Appendix 3

In this Appendix 3, we solve for the initial value of aggregate output $X_i(0)$ when $\vartheta_{i,0} < K_i(0) \leq \vartheta_{i,1}$, and also show how to derive the threshold values for $\vartheta_{i,n}, \forall n = 0, 1, 2, ...$Moreover, we also characterize what happens when country i has already taken off at time 0. Without loss of generality, we demonstrate how to characterize $X_1(0)$ and $\{\vartheta_{1,n}\}_{n=0}^{\infty}$. The values for country 2 can be derived similarly.

Part I. We analyze the case when $K_{1,0} \in (0, \vartheta_{1,0}]$, so country 1 must start by producing good 0 only.

$$\max_{C_1(t)} \int_0^{t_{1,0}} \frac{C_1(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt + \sum_{n=0}^{\infty} \int_{t_{1,n}}^{t_{1,n+1}} \frac{C_1(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt$$

subject to

$$\dot{K_1} = \begin{cases} \xi_1 K_1 & \text{when} \quad 0 \leq t \leq t_{1,0} \\ \xi_1 K_1 - E_{1,(0,1)}(X_1), & \text{when} \quad t_{1,0} \leq t \leq t_{1,1} \\ \xi_1 K_1 - E_{1,(n,n+1)}(X_1), & \text{when} \quad t_{1,n} \leq t \leq t_{1,n+1}, \text{ for } n \geq 1 \end{cases},$$

$K_1(0)$ is given.

When $0 \leq t \leq t_{1,0},$ we must have $X_1(t) = L_1$, so $C_1(t) = \alpha {L_1}^{\alpha} X_2^{\beta}(t)$. The associated discounted-value Hamiltonian with the Lagrangian multipliers is the following

$$H_0 = \frac{C_1(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} + \eta_0 \xi_1 K_1(t) + \zeta_0^0 \left[\alpha {L_1}^{\alpha} X_2^{\beta}(t) - C_1(t) \right].$$

The first order condition and Kuhn-Tucker condition are

$$C_1(t)^{-\sigma} e^{-\rho t} = \zeta_0^0;$$

$$\zeta_0^0 \left[\alpha {L_1}^{\alpha} X_2^{\beta}(t) - C_1(t) \right] = 0;$$

$$\alpha {L_1}^{\alpha} X_2^{\beta}(t) - C_1(t) = 0 \text{ when } \zeta_0^0 > 0,$$

and

$$\dot{\eta_0} = -\frac{\partial H_0}{\partial K_1} = -\eta_0 \xi_1.$$

No capital is used for production before taking off, so

$$\dot{K_1}(t) = \xi_1 K_1(t).$$

When capital stock K_1 exceeds $\vartheta_{1,0}$ by an infinitessimal amount, the economy produces both good 0 and good 1. From that point on, the problem is exactly the same as the one we have just solved in Part I. By definition, $t_{1,0}$ is the time point when K_1 equals $\vartheta_{1,0}$:

$$K_{1,0} e^{\xi_1 t_{1,0}} = \vartheta_{1,0},$$

so $t_{1,0} = \frac{\log \frac{\vartheta_{1,0}}{K_{1,0}}}{\xi_1}$. Consider the simple case when $i^{**} = 1$, so the consumption growth rate and output growth rate will be constant after taking off in country 1.

$$C_1(t) = \begin{cases} \alpha {L_1}^{\alpha} X_2^{\beta}(t), & \text{when} \quad t \leq t_{1,0} \\ \alpha {L_1}^{\alpha} X_2^{\beta}(t) e^{\alpha h_1 (t - t_{1,0})}, & \text{when} \quad t > t_{1,0} \end{cases},$$

where $h_1 = \beta(\xi_1 - \xi_2) + \frac{\alpha\xi_1+\beta\xi_2-\rho}{\sigma}$. Since $t_{1,j}$ is the time point when only good j is produced, for any $j \geq 1$, we have $L_1 e^{h_1(t_{1,j}-t_{1,0})} = X_1(t_{1,j}) = \lambda^j L_1$, so $t_{1,j} = t_{1,0} + \left(\frac{\log\lambda}{h_1}\right) j$, $t_{1,0} = \frac{\log\frac{\vartheta_{1,0}}{K_{1,0}}}{\xi_1}$.

Correspondingly, the capital stock on the equilibrium path is given by

$$K_1(t) = \begin{cases} K_{1,0}e^{\xi_1 t}, & \text{for} \quad t \in [0, t_{1,0}] \\ \frac{-\frac{aL_1}{\lambda-1}}{h_1-\xi_1}e^{h_1(t-t_{1,0})} + \frac{-aL_1}{\xi_1(\lambda-1)} + \left[\vartheta_{1,0} + \frac{\frac{aL_1}{\lambda-1}}{h_1-\xi_1} + \frac{aL_1}{\xi_1(\lambda-1)}\right]e^{\xi_1(t-t_{1,0})}, & \text{for} \quad t \in [t_{1,0}, t_{1,1}] \\ F(t), & \text{for} \quad t \in [t_{1,n}, t_{1,n+1}], \text{ any } n \geq 1 \end{cases}, \tag{58}$$

where

$$F(t) \equiv -\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\left[\frac{L_1 e^{h_1 t}}{(h_1-\xi_1)} + \frac{\lambda^n(a-\lambda)L_1}{\xi_1(a-1)}\right] + [\lambda^n]^{\frac{-\xi}{h_1}}\left\{K_1(t_{1,n}) + \frac{a^{n+1}-a^n}{\lambda-1}L_1\left[\frac{1}{(h_1-\xi_1)} + \frac{(a-\lambda)}{\xi_1(a-1)}\right]\right\}e^{\xi_1(t-t_{1,0})}.$$

By continuity, we have

$$K_1(t_{1,n+1}) = F(t_{1,n+1}) \equiv -\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\left[\frac{L_1 e^{h_1 t_{1,n+1}}}{(h_1-\xi_1)} + \frac{\lambda^n(a-\lambda)L_1}{\xi_1(a-1)}\right] + [\lambda^n]^{\frac{-\xi_1\sigma}{\xi_1-\rho}}\left\{K_1(t_{1,n}) + \frac{a^{n+1}-a^n}{\lambda-1}L_1\left[\frac{1}{(h_1-\xi_1)} + \frac{(a-\lambda)}{\xi_1(a-1)}\right]\right\}e^{\xi_1(t_{1,n+1}-t_{1,0})}$$

$$K(t_{1,n+1}) = \psi_1\lambda^{\frac{n\xi_1}{h_1}}\frac{a\lambda^{\frac{-\xi_1}{h_1}}\left[1-\left(a\lambda^{\frac{-\xi_1}{h_1}}\right)^n\right]}{1-a\lambda^{\frac{-\xi_1}{h_1}}} + \lambda^{\frac{n\xi_1}{h_1}}K_1(t_{1,1})$$

where

$$\psi_1 \equiv \frac{L_1(a-1)}{\lambda-1}\left\{\left(\lambda^{\frac{\xi_1}{h_1}}-\lambda\right)\frac{1}{(h_1-\xi_1)} + \left(\lambda^{\frac{\xi_1}{h_1}}-1\right)\frac{(a-\lambda)}{\xi_1(a-1)}\right\}.$$

The transversality condition is derived from

$$\lim_{t\to\infty} H(t) = 0,$$

so

$$\lim_{t\to\infty}\left[\frac{C_1(t)^{1-\sigma}-1}{1-\sigma}e^{-\rho t} + \eta_{n(t),n(t)+1}\left[\xi_1 K_1(t) - E_{1,(n(t),n(t)+1)}(X_1(t))\right]\right] = 0.$$

Note that

$$\begin{aligned}
&\lim_{t\to\infty}\left[\frac{C_1(t)^{1-\sigma}-1}{1-\sigma}e^{-\rho t}+\eta_{n(t),n(t)+1}\left[\xi_1 K_1(t)-E_{1,(n(t),n(t)+1)}(X_1(t))\right]\right]\\
&=\lim_{t\to\infty}\left[\frac{C_1(0)^{1-\sigma}e^{\frac{(1-\sigma)(\alpha\xi_1+\beta\xi_2)-\rho}{\sigma}t}}{1-\sigma}+\eta_{n(t),n(t)+1}\left[\xi_1 K_1(t)-E_{1,(n(t),n(t)+1)}(X_1(t))\right]\right]\\
&=\lim_{t\to\infty}\eta_{n(t),n(t)+1}[\xi_1 K_1(t)-E_{1,(n(t),n(t)+1)}(X_1(t))]\\
&=\lim_{t\to\infty}\left\{\eta_{(0)}e^{-\xi_1 t}\left[\xi_1 K_1(t)-\left[X_1(0)e^{h_1 t}-\frac{\lambda^{n(t)}(a-\lambda)}{a-1}L_1\right]\frac{a^{n(t)+1}-a^{n(t)}}{\lambda^{n(t)+1}-\lambda^{n(t)}}\right]\right\}\\
&=\lim_{t\to\infty}\left\{\eta_{(0)}\left[\xi_1 K_1(t)e^{-\xi_1 t}-\left[-\frac{e^{-\xi_1 t}\lambda^{n(t)}(a-\lambda)}{a-1}L_1\right]\frac{a^{n(t)+1}-a^{n(t)}}{\lambda-1}\right]\right\}\\
&=\lim_{t\to\infty}K_1(t)e^{-\xi_1 t},
\end{aligned}$$

where the second equality is due to (12), the fourth equality comes from $\xi_1 > h_1$. Thus we must have

$$\lim_{t\to\infty}K_1(t)e^{-\xi_1 t}=0,$$

which implies $\lim_{t\to\infty}F(t)e^{-\xi_1 t}=0$,

$$\begin{aligned}
&\Rightarrow \lim_{n\to\infty}\left(\lambda^{\frac{-\xi_1}{h_1}}a\right)^n\frac{K_1(t_{1,n})}{a^n}=0\\
&\Rightarrow \lim_{n\to\infty}\left(\lambda^{\frac{-\xi_1}{h_1}}\right)^{n+1}\psi_1\lambda^{\frac{n\xi_1}{h_1}}\frac{a\lambda^{\frac{-\xi_1}{h_1}}\left[1-\left(a\lambda^{\frac{-\xi_1}{h_1}}\right)^n\right]}{1-a\lambda^{\frac{-\xi_1}{h_1}}}+\left(\lambda^{\frac{-\xi_1}{h_1}}\right)^{n+1}\lambda^{\frac{n\xi_1}{h_1}}K_1(t_{1,1})=0\\
&\Rightarrow K_1(t_{1,1})=-\psi_1\frac{a\lambda^{\frac{-\xi_1}{h_1}}}{1-a\lambda^{\frac{-\xi_1}{h_1}}}.
\end{aligned}$$

By revoking (58), we obtain

$$\frac{-\frac{aL_1}{\lambda-1}}{h_1-\xi_1}e^{h_1(t_{1,1}-t_{1,0})}+\frac{-aL_1}{\xi_1(\lambda-1)}+\left[\vartheta_{1,0}+\frac{\frac{aL_1}{\lambda-1}}{h_1-\xi_1}+\frac{aL_1}{\xi_1(\lambda-1)}\right]e^{\xi_1(t_{1,1}-t_{1,0})}=-\psi_1\frac{a\lambda^{\frac{-\xi_1}{h_1}}}{1-a\lambda^{\frac{-\xi_1}{h_1}}},$$

which yields

$$\vartheta_{1,0}=\frac{aL_1h_1\left[\lambda^{1-\frac{\xi_1}{h_1}}-1\right]\left(1-\lambda^{\frac{-\xi_1}{h_1}}\right)}{\left(1-a\lambda^{\frac{-\xi_1}{h_1}}\right)(\lambda-1)(h_1-\xi_1)\xi_1}. \tag{59}$$

Using the similar algorithm, we can fully characterize the case when $K_{1,0} > \vartheta_{1,1}$. For country 2, the early bloomer, the analysis is similar. **Q.E.D.**

Part II. We derive the necessary and sufficient condition under which country 1 starts with industries 0 and 1. Again, for simplicity, assume $i^{**}=1$. When $t\in[0,t_{1,1}]$, by Table 1, we have

$$E_1(t) = \frac{a}{\lambda - 1}(X_1(t) - L_1) = \frac{a}{\lambda - 1}(X_1(0)e^{h_1 t} - L_1),$$

where $h_1 = \beta(\xi_1 - \xi_2) + \frac{\alpha\xi_1 + \beta\xi_2 - \rho}{\sigma}$. Correspondingly,

$$\dot{K}_1 = \xi_1 K_1(t) - E_1(C_1(t)) = \xi_1 K_1(t) - \frac{a}{\lambda - 1}(X_1(0)e^{h_1 t} - L_1)$$

Solving this first-order differential equation with the condition $K_1(0) = K_{1,0}$, we obtain

$$K_1(t) = \frac{-\frac{aX_1(0)}{\lambda-1}}{h_1 - \xi_1} e^{h_1 t} + \frac{-aL_1}{\xi_1(\lambda - 1)} + \left[K_{1,0} + \frac{\frac{aX_1(0)}{\lambda-1}}{h_1 - \xi_1} + \frac{aL_1}{\xi_1(\lambda - 1)}\right] e^{\xi_1 t}, \tag{60}$$

which implies

$$K_1(t_{1,1}) = \frac{-\frac{a\lambda L_1}{\lambda-1}}{h_1 - \xi_1} + \frac{-aL_1}{\xi_1(\lambda - 1)} + \left[K_{1,0} + \frac{\frac{aX_1(0)}{\lambda-1}}{h_1 - \xi_1} + \frac{aL_1}{\xi_1(\lambda - 1)}\right] \left(\frac{\lambda L_1}{X_1(0)}\right)^{\frac{\xi_1}{h_1}}. \tag{61}$$

When $t \in [t_{1,n}, t_{1,n+1}]$ for $\forall n \geq 1$, we have

$$K_1(t) = -\frac{a^{n+1} - a^n}{\lambda^{n+1} - \lambda^n}\left[\frac{X_1(0)e^{h_1 t}}{h_1 - \xi_1} + \frac{\lambda^n(a - \lambda)L_1}{\xi_1(a - 1)}\right] + \theta_{n.n+1} e^{\xi_1 t}. \tag{62}$$

which, together with $X_1(0)e^{h_1 t_{1,n}} = X_1(t_{1,n}) = \lambda^n L_1$, determines

$$\theta_{n.n+1} = \left[\frac{\lambda^n L_1}{X_1(0)}\right]^{\frac{-\xi_1}{h_1}} \left\{K_1(t_{1,n}) + \frac{a^{n+1} - a^n}{\lambda - 1} L_1 \left[\frac{1}{h_1 - \xi_1} + \frac{(a - \lambda)}{\xi_1(a - 1)}\right]\right\}. \tag{63}$$

Substituting $t = t_{1,n+1} = \frac{\log \frac{\lambda^{n+1} L_1}{X_1(0)}}{h_1}$ and (63) into (62) yields

$$K_1(t_{1,n+1}) = \lambda^{\frac{\xi_1}{h_1}} K_1(t_{1,n}) + \frac{a^{n+1} - a^n}{\lambda - 1} L_1 \left[\frac{\lambda^{\frac{\xi_1}{h_1}} - \lambda}{h_1 - \xi_1} + \frac{(a - \lambda)(\lambda^{\frac{\xi_1}{h_1}} - 1)}{\xi_1(a - 1)}\right],$$

which can be used recursively to obtain

$$K_1(t_{1,n}) = \lambda^{\frac{(n-1)\xi_1}{h_1}} K_1(t_{1,1}) + (a - 1)B\lambda^{\frac{(n-2)\xi_1}{h_1}} \frac{a\left[1 - \left(a\lambda^{\frac{-\xi_1}{h_1}}\right)^{n-1}\right]}{1 - a\lambda^{\frac{-\xi_1}{h_1}}}, \text{ for any } n \geq 2 \tag{64}$$

where parameter B is defined as

$$B \equiv \frac{L_1}{\lambda - 1}\left[\frac{\lambda^{\frac{\xi_1}{h_1}} - \lambda}{h_1 - \xi_1} + \frac{(a - \lambda)\left(\lambda^{\frac{\xi_1}{h_1}} - 1\right)}{\xi_1(a - 1)}\right].$$

(20) implies $B < 0$. Substituting (62), (63) and (64) into the transversality condition $\lim_{t\to\infty} K_1(t)e^{-\xi_1 t} = 0$ and by revoking (20), we obtain

$$\lambda^{\frac{-\xi_1}{h_1}} K_1(t_{1,1}) + (a-1)B\lambda^{-2\frac{\xi_1}{h_1}} \frac{a}{1-a\lambda^{\frac{-\xi_1}{h_1}}} = 0,$$

which yields

$$K_1(t_{1,1}) = -\frac{(a-1)B\lambda^{\frac{-\xi_1}{h_1}} a}{1-a\lambda^{\frac{-\xi_1}{h_1}}} > 0. \tag{65}$$

It can be verified that, without condition (20), the transversality condition cannot hold unless both B and $K_1(t_{1,1})$ are equal to zero, which is economically unreasonable because $K_1(t_{1,1}) > 0$ must hold due to the resource contraint as no international borrowing or lending is allowed.

According to (61), we have

$$\begin{aligned}&\left[K_{1,0} + \frac{aX_1(0)}{(\lambda-1)(h_1-\xi_1)} + \frac{aL_1}{\xi_1(\lambda-1)}\right]\left(\frac{\lambda L_1}{X_1(0)}\right)^{\frac{\xi_1}{h_1}} \\ &= \frac{a\lambda L_1}{(\lambda-1)(h_1-\xi_1)} + \frac{aL_1}{\xi_1(\lambda-1)} - \frac{(a-1)B\lambda^{\frac{-\xi_1}{h_1}} a}{1-a\lambda^{\frac{-\xi_1}{h_1}}}.\end{aligned} \tag{66}$$

Observe that the right hand side is strictly positive and that the left hand side is a strictly decreasing function of $X_1(0)$, so we can uniquely pin down $X_1(0)$.

(65) implies that $K(t_{1,1})$ does not depend on $K_1(0)$, so (64) indicates that $K_1(t_{1,n})$ is independent of $K_1(0)$ for all $n \geq 1$ conditional on that country 1 is the late bloomer. To ensure that good 0 and good 1 are produced at time 0, we must impose $L_1 < X_1^*(0) \leq \lambda L_1$.

To ensure $X_1^*(0) \leq \lambda L_1$, from (66), it requires

$$K_{1,0} \leq \vartheta_{1,1} \equiv K_1(t_{1,1}) = -\frac{a\lambda^{\frac{-\xi_1}{h_1}}}{1-a\lambda^{\frac{-\xi_1}{h_1}}} \frac{L_1}{\lambda-1}\left[\frac{\xi_1\left(a-\lambda^{\frac{\xi_1}{h_1}}\right)(1-\lambda) + h_1(a-\lambda)\left(\lambda^{\frac{\xi_1}{h_1}}-1\right)}{(h_1-\xi_1)\xi_1}\right],$$

which is strictly positive due to (20). To ensure $X_1^*(0) > L_1$, by revoking (66), we must require

$$K_{1,0} > \frac{a}{\left(1-a\lambda^{\frac{-\xi_1}{h_1}}\right)(\lambda-1)} \frac{h_1 L_1}{(h_1-\xi_1)\xi_1}\left[\frac{\left(1-\lambda^{1-\frac{\xi_1}{h_1}}\right)(1-\lambda^{\frac{\xi_1}{h_1}})}{\lambda^{\frac{\xi_1}{h_1}}}\right].$$

Observe that the right hand side is nothing but $\vartheta_{1,0}$ given by (59)!

Since $K_1(t_{1,1})$ is known (given by (65)), $K_1(t_{1,n})$ is uniquely determined by (64) for any $n \geq 2$. Consequently, for any $t \geq 0$, $K(t)$ can be explicitly computed from (60) or

(62) and (63), where $t_{i,n} = \frac{\log \frac{\lambda^n L_1}{X_1(0)}}{h_1}$ for any $n \geq 0$ because $X_1^*(0)$ is uniquely determined by (66).

Using (62) and (60), we obtain for $i = i^{**}$,

$$K_i(t) = \begin{cases} \frac{-\frac{aX_i(0)}{\lambda-1}}{h_i-\xi_i} e^{h_{ii}t} + \frac{-aL_i}{\xi_i(\lambda-1)} + \left[K_{i0} + \frac{\frac{aX_i(0)}{\lambda-1}}{h_i-\xi_i} + \frac{aL_i}{\xi_i(\lambda-1)}\right] e^{\xi_i t} & \text{when} \quad t \in [0, t_{i,1}] \\ \alpha_{i,n} + \beta_{i,n} e^{h_1 t} + \gamma_{i,n} e^{\xi_i t} & \text{when} \quad t \in [t_{i,n}, t_{i,n+1}], \text{ for any } n \geq 1 \end{cases} \tag{67}$$

where $t_{i,n} = \frac{\log \frac{\lambda^n L_1}{X_1(0)}}{h_1}$ and for any $n \geq 1$,

$$\alpha_{i,n} = -\frac{a^n(a-\lambda)L_i}{\xi_i(\lambda-1)},$$
$$\beta_{i,n} = -\left(\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\right)\frac{X_i(0)}{(h_i-\xi_i)},$$
$$\gamma_{i,n} = \left[\frac{\lambda^n L_i}{X_i(0)}\right]^{\frac{-\xi_i}{h_i}} \left\{\vartheta_{i,n} + \frac{(a^{n+1}-a^n)L_i}{\lambda-1}\left[\frac{1}{(h_i-\xi_i)} + \frac{(a-\lambda)}{\xi_i(a-1)}\right]\right\}.$$

Note that

$$\vartheta_{i,1} \equiv K_i(t_{i,1}) = \frac{-\frac{a\lambda L}{\lambda-1}}{h_i-\xi_i} + \frac{-aL_i}{\xi_i(\lambda-1)} + \left[K_{i0} + \frac{\frac{aX_i(0)}{\lambda-1}}{h_i-\xi_i} + \frac{aL_i}{\xi_i(\lambda-1)}\right]\left(\frac{\lambda L_i}{X_i(0)}\right)^{\frac{\xi_i}{h_i}},$$

and $\{\vartheta_{i,n}\}_{n=2}^{\infty}$ are all constants, and $\vartheta_{i,n} \equiv K_i(t_{i,n})$ can be sequentially computed by applying (67) recursively with $K_i(t_{i,n-1})$ known. The initial output $X_1(0)$ is uniquely determined by (66) obtained from the transversality condition, $X_2(0)$ can be obtained using the same method.

By revoking Table 1, it is easy to show that when $K_i(0) \in (\vartheta_{i,0}, \vartheta_{i,1})$, where $i = i^{**}$, both good 0 and good 1 are produced at time 0, the output of each industry is given by

$$x_{i,n}^*(t) = \begin{cases} \frac{X_i(0)e^{h_i t}}{\lambda^n-\lambda^{n-1}} - \frac{L_i}{\lambda-1} & \text{when} \quad t \in [t_{i,n-1}, t_{i,n}] \\ -\frac{X_i(0)e^{h_i t}}{\lambda^{n+1}-\lambda^n} + \frac{\lambda L_i}{\lambda-1}, & \text{when} \quad t \in [t_{i,n}, t_{i,n+1}] \\ 0, & \text{otherwise} \end{cases}, \text{ for all } n \geq 2$$

$$x_{i,1}^*(t) = \begin{cases} \frac{X_i(0)e^{h_i t}-L_i}{\lambda-1}, & \text{when} \quad t \in [0, t_{i,1}] \\ -\frac{X_i(0)e^{h_i t}}{\lambda^2-\lambda} + \frac{\lambda L_i}{\lambda-1}, & \text{when} \quad t \in [t_{i,1}, t_{i,2}] \\ 0, & \text{otherwise} \end{cases},$$

$$x_{i,0}^*(t) = \begin{cases} L_i - \frac{X_i(0)e^{h_i t}-L_i}{\lambda-1}, & \text{when} \quad t \in [0, t_{i,1}] \\ 0, & \text{otherwise} \end{cases},$$

where $h_i = \widetilde{h}(i)$, as specified by (18). Graphically, it means that the diversification cone for good 0 and good 1 is "truncated", as shown in Figure 5.

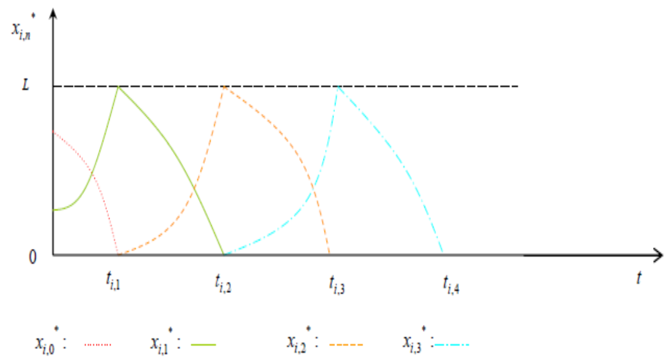

Figure 5. Industrial Dynamics in Country i with International Trade when $t_{i,0} = 0$ and $t_{i,1} > 0$.

Similarly, when $K_i(0)$ is such that both good $\widetilde{n}$ and good $\widetilde{n}$ +1 are produced at time 0 for some $\widetilde{n} \geq 1$, then the industrial dynamics is given by

$$x^*_{i,n}(t) = \begin{cases} \frac{X_i(0)e^{h_i t}}{\lambda^n - \lambda^{n-1}} - \frac{L_i}{\lambda - 1} & \text{when} \quad t \in [t_{i,n-1}, t_{i,n}] \\ -\frac{X_i(0)e^{h_i t}}{\lambda^{n+1} - \lambda^n} + \frac{\lambda L_i}{\lambda - 1}, & \text{when} \quad t \in [t_{i,n}, t_{i,n+1}] \\ 0, & \text{otherwise} \end{cases}, \text{ for all } n \geq \widetilde{n} + 2$$

$$x^*_{i,\widetilde{n}+1}(t) = \begin{cases} \frac{X_i(0)e^{h_i t}}{\lambda^{\widetilde{n}+1} - \lambda^{\widetilde{n}}} - \frac{L_i}{\lambda - 1}, & \text{when} \quad t \in [0, t_{i,\widetilde{n}+1}] \\ -\frac{X_i(0)e^{h_i t}}{\lambda^{\widetilde{n}+2} - \lambda^{\widetilde{n}+1}} + \frac{\lambda L_i}{\lambda - 1}, & \text{when} \quad t \in [t_{i,\widetilde{n}+1}, t_{i,\widetilde{n}+2}] \\ 0, & \text{otherwise} \end{cases},$$

$$x^*_{i,\widetilde{n}}(t) = \begin{cases} -\frac{X_i(0)e^{h_i t}}{\lambda^{\widetilde{n}+1} - \lambda^{\widetilde{n}}} + \frac{\lambda L_i}{\lambda - 1}, & \text{when} \quad t \in [0, t_{i,\widetilde{n}+1}] \\ 0, & \text{otherwise} \end{cases},$$

$$x^*_{i,n}(t) = 0 \text{ for any } t \geq 0 \text{ and any } n \leq \widetilde{n} - 1.$$

The corresponding industry dynamics can be illustrated graphically in the following graph.

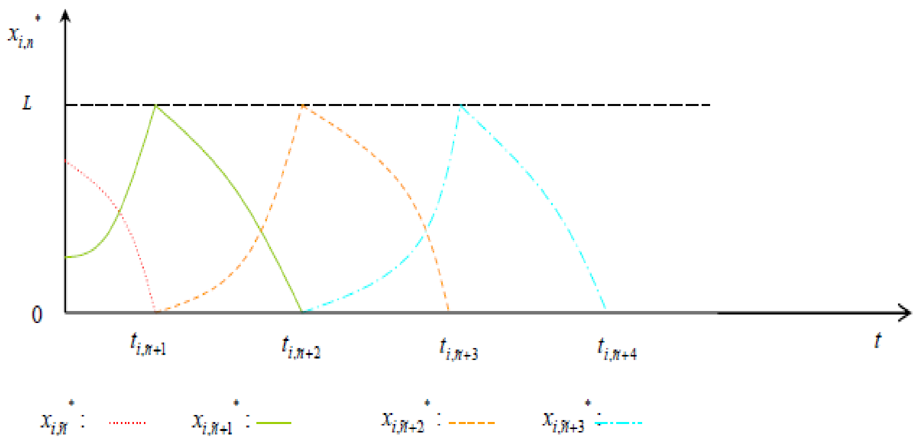

Figure 6. Industrial Dynamics in Country i with International Trade when $t_{i,\widetilde{n}} = 0$ and $t_{i,\widetilde{n}+1} > 0$ for some $\widetilde{n} \geq 1$.

Part III. The previous derivations in Part I and Part II have already proved the following proposition, which states how the initial industries and $X_i(0)$ are determined for the late bloomer $i = i^{**} \in \{1, 2\}$. For the convenience of exposition, define

$$\widetilde{B}_i \equiv \frac{L_i}{\lambda - 1}\left[\frac{\lambda^{\frac{\xi_i}{h_i}} - \lambda}{h_i - \xi_i} + \frac{(a-\lambda)\left(\lambda^{\frac{\xi_i}{h_i}} - 1\right)}{\xi_i(a-1)}\right] < 0. \tag{68}$$

Proposition 16 *Suppose $i = i^{**}$. [1] When $K_{i,0} \in (0, \vartheta_{i,0}]$. The initial output of country i is*

$$X_i(0) = L_i,$$

and the capital accumulation function is

$$K_i(t) = \begin{cases} K_{i,0}e^{\xi_i t}, & \textit{for} \quad t \in [0, t_{i,0}] \\ \frac{-\frac{aL_i}{\lambda-1}}{h_i - \xi_i}e^{h_i t} + \frac{-aL_i}{\xi_i(\lambda-1)} + \left[\vartheta_{i,0} + \frac{\frac{aL_i}{\lambda-1}}{h_i-\xi_i} + \frac{aL_i}{\xi_i(\lambda-1)}\right]e^{\xi_i t} & \textit{for} \quad t \in [t_{i,0}, t_{i,1}] \\ \alpha_{i,n} + \beta_{i,n}e^{h_i t} + \gamma_{i,n}e^{\xi_i t}, & \textit{for} \quad t \in [t_{i,n}, t_{i,n+1}], \textit{ any } n \geq 1 \end{cases};$$

[2]when $K_{i,0} \in (\vartheta_{i,0}, \vartheta_{i,1}]$, $X_i(0)$ is uniquely determined by

$$\left[K_{i,0} + \frac{aL_i}{\xi_i(\lambda-1)} + \frac{aX_i(0)}{(\lambda-1)(h_i-\xi_i)}\right]\left(\frac{\lambda L_i}{X_i(0)}\right)^{\frac{\xi_i}{h_i}}$$
$$= \frac{aL_i}{\xi_i(\lambda-1)} + \frac{a\lambda L_i}{(\lambda-1)(h_i-\xi_i)} - \frac{a(a-1)\widetilde{B}_i\lambda^{\frac{-\xi_i}{h_i}}}{1 - a\lambda^{\frac{-\xi_i}{h_i}}};$$

and

$$K_i(t) = \begin{cases} \frac{-\frac{aX_i(0)}{\lambda-1}}{h_i-\xi_i} e^{h_{ii}t} + \frac{-aL_i}{\xi_i(\lambda-1)} + \left[K_{i0} + \frac{\frac{aX_i(0)}{\lambda-1}}{h_i-\xi_i} + \frac{aL_i}{\xi_i(\lambda-1)}\right] e^{\xi_i t} & \textit{when} \quad t \in [0, t_{i,1}] \\ \alpha_{i,n} + \beta_{i,n} e^{h_i t} + \gamma_{i,n} e^{\xi_i t} & \textit{when} \quad \begin{matrix} t \in [t_{i,n}, t_{i,n+1}] \\ \textit{for any } n \geq 1 \end{matrix} \end{cases};$$

[3]when $K_{i,0} \in (\vartheta_{i,m}, \vartheta_{i,m+1}]$, *for any* $m \geq 1$, $X_i(0)$ *is uniquely determined by*

$$\left[K_{i,0} + \frac{a^m(a-\lambda)L_i}{\xi_i(\lambda-1)} + \frac{a^{m+1}-a^m}{\lambda^{m+1}-\lambda^m}\frac{X_i(0)}{(h_i-\xi_i)}\right]\left[\frac{\lambda^{m+1}L_i}{X_i(0)}\right]^{\frac{\xi_i}{h_i}}$$
$$= \frac{a^m(a-\lambda)L_i}{\xi_i(\lambda-1)} + \left(\frac{a-1}{\lambda-1}\right)\frac{a^m\lambda L_i}{(h_i-\xi_i)} - \frac{a(a-1)\widetilde{B}_i\lambda^{\frac{-\xi_i}{h_i}}}{1-a\lambda^{\frac{-\xi_i}{h_i}}},$$

and

$$K_i(t) = \begin{cases} \alpha_{i,m} + \beta_{i,m}e^{h_i t} + \left[K_i(0) + \frac{a^m(a-\lambda)L_i}{\xi_i(\lambda-1)} + \frac{a^{m+1}-a^m}{\lambda^{m+1}-\lambda^m}\frac{X_i(0)}{(h_i-\xi_i)}\right]e^{\xi_i t}, & \textit{when} \quad t \in [0, t_{i,m+1}] \\ \alpha_{i,n} + \beta_{i,n}e^{h_i t} + \gamma_{i,n}e^{\xi_i t}, & \textit{when} \quad \begin{matrix} t \in [t_{i,n}, t_{i,n+1}] \\ \textit{for any } n \geq m+1 \end{matrix} \end{cases},$$

where $\widetilde{B}_i$ *is given by (68),* $\alpha_{i,n}, \beta_{i,n}$ *and* $\gamma_{i,n}$ *are defined by (28)-(30), respectively,* $t_{i,n}$ *is given by Proposition 8; and* $\vartheta_{i,n}$ *is given by Proposition 7 for any* $n \geq 0$.

Appendix 4. Proof of Lemma 10.

The budget constraint for a representative household in country 1 is $P_1C_{1,1} + P_2(1+\tau_2)C_{1,2} = P_1X_1 + T_1$. Utility function (2) implies $C_{11} = \alpha(X_1 + \frac{T_1}{P_1})$ and $C_{12} = \frac{\beta(P_1X_1+T_1)}{P_2(1+\tau_2)}$. Similarly, country 2 household's budget constraint is $P_1(1+\tau_1)C_{2,1} + P_2C_{2,2} = P_2X_2 + T_2$.Thus we must have $C_{21} = \alpha\frac{P_2X_2+T_2}{P_1(1+\tau_1)}; C_{22} = \beta(X_2 + \frac{T_2}{P_2})$. In the equilibrium, the tariff revenues are $T_1 = \frac{\beta\tau_2P_1X_1}{(1+\alpha\tau_2)}$ and $T_2 = \frac{\alpha\tau_1P_2X_2}{(1+\beta\tau_1)}$. Plugging all these into the market clearing conditions for good 1 and good 2 yields

$$\alpha(X_1 + \frac{P_2\tau_2C_{1,2}}{P_1}) + \alpha\frac{P_2X_2 + P_1\tau_1C_{2,1}}{P_1(1+\tau_1)} = X_1,$$
$$\frac{\beta\left(P_1X_1 + P_2\tau_2C_{1,2}\right)}{P_2(1+\tau_2)} + \beta(X_2 + \frac{P_1\tau_1C_{2,1}}{P_2}) = X_2,$$

which imply

$$C_{11} = \frac{\alpha(1+\tau_2)X_1}{(1+\alpha\tau_2)};\ C_{12} = \frac{\alpha X_2}{(1+\beta\tau_1)}.$$
$$C_{21} = \frac{\beta X_1}{(1+\alpha\tau_2)};\ C_{22} = \frac{(\tau_1+1)\beta X_2}{1+\beta\tau_1}.$$

Then (31) and (32) are obtained naturally. (33) can be derived easily. Observe that the decentralized production decisions in each country remain unaffected by international trade in this static economy, so X_1 and X_2 are exactly given in Table 1. **Q.E.D.**

Appendix 5. Proof of Lemma 11.

Proof. By following the same method as in Section 3, we establish the following Hamiltonian equation:

$$H_{n,n+1} = \frac{C_1(t)^{1-\sigma}-1}{1-\sigma}e^{-\rho t} + \eta_{n,n+1}\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\left[\xi_1 K(t) - \left[\begin{array}{c}\left[\frac{C_1(t)(1+\alpha\tau_2)^\alpha(1+\beta\tau_1)^\beta}{\alpha X_2^\beta(t)(1+\tau_2)^\alpha}\right]^{\frac{1}{\alpha}} \\ -\frac{\lambda^n(a-\lambda)}{a-1}L\end{array}\right]\right].$$

Using the first order conditions, we obtain

$$C_1(t)^{-\sigma}e^{-\rho t} = \eta_{n,n+1}\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\frac{1}{\alpha}C_1(t)^{\frac{1}{\alpha}-1}\left[\frac{(1+\alpha\tau_2)^\alpha(1+\beta\tau_1)^\beta}{\alpha X_2^\beta(t)(1+\tau_2)^\alpha}\right]^{\frac{1}{\alpha}},$$

which yields

$$(\frac{1}{\alpha}+\sigma-1)\frac{\overset{\bullet}{C_1}(t)}{C_1(t)} = \xi_1 - \rho + \frac{\beta}{\alpha}\frac{\overset{\bullet}{X_2}(t)}{X_2(t)} - \frac{\alpha\dot{\tau}_2}{1+\alpha\tau_2} - \frac{\beta}{\alpha}\frac{\beta\dot{\tau}_1}{1+\beta\tau_1} + \frac{\dot{\tau}_2}{\tau_2+1}.$$

Similarly, for country 2, we have

$$H_{m,m+1} = \frac{C_2(t)^{1-\sigma}-1}{1-\sigma}e^{-\rho t}+\eta_{m,m+1}\left[\xi_2 K(t) - \frac{a^{m+1}-a^m}{\lambda^{m+1}-\lambda^m}\left[\begin{array}{c}\left[\frac{C_1(t)(1+\alpha\tau_2)^\alpha(1+\beta\tau_1)^\beta}{\beta X_1^\alpha(t)(1+\tau_1)^\beta}\right]^{\frac{1}{\beta}} \\ -\frac{\lambda^m(a-\lambda)}{a-1}L\end{array}\right]\right],$$

which gives

$$(\frac{1}{\beta}+\sigma-1)\frac{\overset{\bullet}{C_2}(t)}{C_2(t)} = \xi_2 - \rho + \frac{\alpha}{\beta}\frac{\overset{\bullet}{X_1}(t)}{X_1(t)} - \frac{\alpha}{\beta}\frac{\alpha\dot{\tau}_2}{1+\alpha\tau_2} - \frac{\beta\dot{\tau}_1}{1+\beta\tau_1} + \frac{\dot{\tau}_1}{\tau_1+1}.$$

Moreover, (34) implies

$$\frac{\overset{\bullet}{C_1}(t)}{C_1(t)} - \frac{\overset{\bullet}{C_2}(t)}{C_2(t)} = \frac{\alpha\dot{\tau}_2}{1+\tau_2} - \frac{\beta\dot{\tau}_1}{1+\tau_1}.$$

(31) implies

$$\frac{\overset{\bullet}{C_1}(t)}{C_1(t)} = \frac{\alpha\dot{\tau}_2}{1+\tau_2} - \alpha\frac{\alpha\dot{\tau}_2}{1+\alpha\tau_2} - \beta\frac{\beta\dot{\tau}_1}{1+\beta\tau_1} + \alpha\frac{\overset{\bullet}{X_1}(t)}{X_1(t)} + \beta\frac{\overset{\bullet}{X_2}(t)}{X_2(t)},$$

and (32) implies

$$\frac{\overset{\bullet}{C_2}(t)}{C_2(t)} = \frac{\beta\dot{\tau}_1}{1+\tau_1} - \alpha\frac{\alpha\dot{\tau}_2}{1+\alpha\tau_2} - \beta\frac{\beta\dot{\tau}_1}{1+\beta\tau_1} + \alpha\frac{\overset{\bullet}{X_1}(t)}{X_1(t)} + \beta\frac{\overset{\bullet}{X_2}(t)}{X_2(t)}.$$

Solving these equations gives (36)-(39). **Q.E.D**.

Appendix 6: Asymmetric C-D aggregation Our previous analysis assumes that the two countries have the same expenditure shares on the two aggregate goods. Now we allow these shares to be country-specific. Suppose

$$C_i = C_{i,1}^{\alpha_i} C_{i,2}^{\beta_i},\ \alpha_i \geq 0, \beta_i \geq 0, \alpha_i + \beta_i = 1 \text{ for } i \in \{1, 2\}. \tag{69}$$

Also assume $\alpha_1 \geq \alpha_2$ to capture the home bias effect. In the long-run dynamic free-trade equilibrium, we have

$$h_1(t) = \frac{(1-\sigma)\left[\rho\beta_2 - \beta_2\xi_1 + \beta_1(\xi_2 - \rho)\right] + \xi_1 - \rho}{\sigma\left[\alpha_2 + \beta_1(1-\sigma) + (1-\alpha_2)\sigma\right]}; \tag{70}$$

$$h_2(t) = \frac{\rho(\alpha_1 - \alpha_2)(1-\sigma) - \rho + (1-\sigma)(\alpha_2\xi_1 - \alpha_1\xi_2) + \xi_2}{\sigma\left[\alpha_2 + \beta_1 + (\alpha_1 - \alpha_2)\sigma\right]}; \tag{71}$$

$$\theta_1(t) = \frac{\left[\alpha_1 - (1-\sigma)(\alpha_1 - \alpha_2)\right]\xi_1 + \beta_1\xi_2 - \rho\left[1 - (1-\sigma)(\alpha_1 - \alpha_2)\right]}{\sigma\left[\alpha_2 + \beta_1 + (\alpha_1 - \alpha_2)\sigma\right]}; \tag{72}$$

$$\theta_2(t) = \frac{\alpha_2\xi_1 + \left[\beta_2 - (1-\sigma)(\alpha_1 - \alpha_2)\right]\xi_2 - \rho\left[1 - (1-\sigma)(\alpha_1 - \alpha_2)\right]}{\sigma\left[\alpha_2 + \beta_1 + (\alpha_1 - \alpha_2)\sigma\right]}. \tag{73}$$

When $\alpha_1 = \alpha_2$ (therefore $\beta_1 = \beta_2$), the above equations degenerate to (16)-(15). It can be verified that, again, consumption growth rates are strictly increasing in both the domestic and foreign ISTP parameters. In addition, the output growth increases with the trade partner's ISTP rate if and only if the inter-temporal elasticity of substitution exceeds one. So the key results remain valid in this more general setting.

Appendix 7: General CES Armington Aggregation

The Cobb-Douglas Armington aggregation assumed in the main text implies that the expenditure shares are fixed and unaffected by any trade policies, which may not be realistic. Suppose the Armington trade assumption in (1) is changed to the general CES function, raising tariff typically leads to a decrease in the expenditure share on imports. This is, for example, supported by Eaton and Kortum (2001), where the expenditure shares are endogenously affected by trade barriers. We adopt a reduced-form approach here by simply assuming $\beta_1'(\tau_2) < 0$, that is, an increase in the tariff on imports from country 2 leads to a decrease in country 1's expenditure share on total imports. Also assume $\alpha_2'(\beta_1) > 0$ to capture the usual general equilibrium effect that, when a country imports more (hence consumes a larger fraction of the income on imports), its trade partner will import more as well because a larger fraction of its own output is now exported. We can easily obtain the following

$$\frac{\partial\theta_1(t)}{\partial\beta_1} \propto (\xi_2-\xi_1)\left[\alpha_2+(1-\alpha_2)\,\sigma-(1-\sigma)\,\beta_1\alpha_2'(\beta_1)\right],$$
$$\frac{\partial^2\theta_1(t)}{\partial\beta_1^2} \propto (\xi_2-\xi_1)(\sigma-1)\left[\alpha_2'(\beta_1)+\beta_1\alpha_2''(\beta_1)\right],$$
$$\frac{\partial h_1(t)}{\partial\beta_1} \propto (1-\sigma)(\xi_2-\xi_1)\left[\alpha_2+(1-\alpha_2)\,\sigma-(1-\sigma)\,\beta_1\alpha_2'(\beta_1)\right],$$
$$\frac{\partial^2 h_1(t)}{\partial\beta_1^2} \propto -(1-\sigma)^2(\xi_2-\xi_1)\left[\alpha_2'(\beta_1)+\beta_1\alpha_2''(\beta_1)\right].$$

First of all, this result immediately means that, when the two countries have the same technical change rate ($\xi_1=\xi_2$) or when $\alpha_2+(1-\alpha_2)\,\sigma-(1-\sigma)\,\beta_1\alpha_2'(\beta_1)=0$, neither the consumption growth rate nor the output growth rate is affected by the change in the expenditure share on imports.

Second, when $\xi_1\neq\xi_2$ and $\alpha_2+(1-\alpha_2)\,\sigma-(1-\sigma)\,\beta_1\alpha_2'(\beta_1)\neq 0$, the consumption growth rate and the output growth rate will change with the import share in the same direction when $\sigma\in(0,1)$, but in the opposite direction when $\sigma\in(1,\infty)$. When $\sigma=1$, the output growth rate does not depend on the import share, but the consumption growth rate may either increase with the import share or decrease with it, depending on whether the foreign technology parameter is larger than the domestic one or not.

More specifically, suppose $\xi_1>\xi_2$ and $\sigma\in(1,\infty)$. The consumption growth rate strictly decreases with the import share ($\frac{\partial\theta_1(t)}{\partial\beta_1}<0$) while the output growth strictly increases with it ($\frac{\partial h_1(t)}{\partial\beta_1}>0$). Together with $\beta_1'(\tau_2)<0$, this result states that when the inter-temporal elasticity of substitution is smaller than one, a tariff reduction will lead to an increase in the output growth but a decrease in the consumption growth for the country that has a larger capital goods production efficiency than its trade partner. The opposite is true for its trade partner. To understand the intuition, observe that there are several competing effects working in the opposite directions following a tariff reduction. First, since a larger fraction of consumption expenditure will be on foreign imports, the saving decision of country 1 will respond more to the inter-temporal change in the imports. Since $\xi_2<\xi_1$, imports become increasingly more expensive relative to its own output, which has two effects. One is the inter-temporal substitution effect which tends to substitute future consumption for today, hence lower the saving rate and output growth rate. The second effect is the negative income effect. The output revenue decreases and therefore consumers tend to lower the consumption and save more, which tends to increase the output growth. But the first effect always dominates the second effect, so the net effect is to lower the output growth. The third effect comes from the export market expansion for the output in country 1 as captured by $\alpha_2'(\beta_1)>0$. It tends to increase the domestic income level. However, since $\xi_2<\xi_1$, the market in country 2 grows more slowly, the export revenues also increase more slowly, which tends to raise the current saving and lower consumption, so the output growth rate increases.

The inter-temporal substitution effect is dominated when the inter-temporal elasticity of substitution is less than 1, so the output growth rate ultimately increases with the tariff reduction. Since the imports occupy a larger share of total consumption and imports grow relatively slowly as $\xi_2 < \xi_1$, the consumption growth rate is decreasing. All these results will be reversed when $\xi_1 < \xi_2$.

By contrast, now suppose $\sigma \in (0, 1)$ while we continue to assume that $\xi_1 > \xi_2$. In this case, both the consumption growth rate and the output growth rate increase with the import share when $\beta_1 \alpha_2'(\beta_1) > \alpha_2(\beta_1) + \frac{\sigma}{1-\sigma}$, but the opposite is true when $\beta_1 \alpha_2'(\beta_1) < \alpha_2(\beta_1) + \frac{\sigma}{1-\sigma}$. In other words, both the consumption growth rate and the output growth reach a local maximum when

$$\beta_1 \alpha_2'(\beta_1) = \alpha_2(\beta_1) + \frac{\sigma}{1-\sigma}. \tag{74}$$

In addition, suppose $\phi(\beta_1) > 1$, $\forall \beta_1 \in (0, 1)$, where $\phi(\beta_1) \equiv -\frac{\beta_1 \alpha_2''(\beta_1)}{\alpha_2'(\beta_1)}$. Economically, $\phi(\beta_1)$ is the elasticity of marginal change in country 2's expenditure share on imports relative to the change in country 1's expenditure share on imports. Then both the consumption growth rate and the output growth rate are strictly concave functions of the import share β_1. Furthermore, suppose $\alpha_2(\beta_1)$ satisfies the following Inada-like condition:

$$\lim_{\beta_1 \to 0} \beta_1 \alpha_2'(\beta_1) > \alpha_2(\beta_1) + \frac{\sigma}{1-\sigma} > \alpha_2'(1), \tag{75}$$

then both $h_1(t)$ and $\theta_1(t)$ reach the unique global maximum when $\beta_1 = \beta_1^*$, where β_1^* is the unique solution to (74) and $\frac{\partial \beta_1^*}{\partial \sigma} < 0$. Suppose β_1 can reach any value in the interval $(0, 1)$ by choosing some finite (possibly negative) τ_2 as $\beta_1'(\tau_2) < 0$, the result means that there exists a finite non-zero tariff (subsidy) rate at which both the consumption and output growth rates are the largest. Furthermore, the larger the inter-temporal elasticity of substitution, the smaller the growth-maximizing tariff rate.